ABSTRACT

Title of Dissertation	MIDDLE GRADES PRE-SERVICE TEACHERS’ TASK SELECTION IN A MEDIATED FIELD EXPERIENCE METHODS COURSE
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Task selection is a critical element of mathematics teaching because mathematical tasks differ in the mathematical opportunities made available to students. Thus, it is important to examine both the tasks that PSTs chose and why they were chosen. This study examines the task selection of 10 pre-service teachers (PSTs) in a middle grades mathematics methods course. Each week, PSTs prepared and delivered 90-minute lessons for their assigned small group of middle grades students in an after-school enrichment program a local middle school. PSTs were free to choose the content of their lesson plans.

I use Remillard’s (2005) framework of the teacher-curriculum relationship paired with a documental approach to didactics to infer PSTs’ instructional aims and their personal and pedagogical resources leveraged during task selection and lesson planning. PSTs' lesson plans, lesson reflections, and semi-structured interviews were analyzed to identify the intellectual resources, perspectives, and epistemologies employed by PSTs when preparing their lessons.

Three broad instructional aims shared by PSTs are identified. For each of the three themes, I describe the shared aim and demonstrate how it combines with other personal resources to form a scheme of utilization which informs PSTs' participation with instructional resources. First, the enrichment sessions should be fun. PSTs differed in how they conceptualized fun, attending to either the structure or the mathematics of the tasks. Second, PSTs aimed to avoid surprises during their lessons by anticipating student responses. Finally, PSTs aimed to select or create tasks that "fit" their students. PSTs assessed task fit by the absence of unproductive struggle and whether students completed the task.

This study identifies several productive beliefs and dispositions held by PSTs when selecting tasks in an early field experience. These beliefs and dispositions can be leveraged by teacher educators to support the development of ambitious teaching practices. Furthermore, this study demonstrates the importance of modeling high cognitive demand tasks in both mathematics methods and content courses.

MIDDLE GRADES PRE-SERVICE TEACHERS' TASK SELECTION IN A MEDIATED FIELD EXPERIENCE METHODS COURSE

By

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Dedication

For my sisters, Maureen and Merideth

growth in our own time
three sisters in harmony
corn, beans and squash

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Chapter 1: Introduction

The selection and implementation of mathematical tasks continues to be a critical aspect of mathematics education and research (Tekkumru-Kisa, Stein & Doyle, 2020). The National Council of Teachers of Mathematics (NCTM) identifies "implementing tasks that promote reasoning and problem solving" as an essential practice of effective mathematics teaching (NCTM, 2014). Teachers must regularly select and implement tasks that provide students with opportunities to actively engage in reasoning, sense-making, and problem solving (NCTM, 2014). More broadly, national teacher accreditation standards expect early career teachers to be equipped to select materials and create learning experiences that invite students to demonstrate their knowledge in a variety of ways (CAEP, 2020; CCSSO, 2013). At a local level, the Commission on Innovation and Excellence in Education (Kirwan Commission, 2019) states that teachers should be equipped to "recognize and effectively use high-quality instructional materials (including online) and to adapt existing curriculum to make it stronger using standards-aligned tools to assist them" (2019, p. 49). At both a national and state level, mathematics teachers are expected to be curriculum curators. Thus, teachers must be equipped with the skills to seek out, identify, select, modify, and implement mathematically rigorous tasks. These skills are especially critical, considering teachers may or may not have access to high quality curriculum materials through their schools or districts (NCTM, 2014).

This research study examines the task selection and lesson planning practices of pre-service middle grades mathematics and science teachers (PSTs) and how their experiences in an undergraduate mediated field experience mathematics methods course[1] supported their development of these practices. To further motivate this research, I begin by sharing my journey to becoming a mathematics teacher educator, which is rooted in my experiences as a student in the mathematics methods course that serves as the context of this study. I then present the research questions, the significance, and contributions of this study.

Researcher Narrative

I begin this research with a personal narrative to provide transparency about my relationship with the course context under investigation. Over a decade ago, I was enrolled in the middle grades mathematics methods course that serves as the context of this study. When I began the course, I was a secondary mathematics education major and planned to teach high school mathematics. By the end of the course, I knew I wanted to teach middle grades students and I changed my major to mathematics, planning to enroll in the Masters' certification program at the same University. Thus, this course was the catalyst in my decision to teach middle grades mathematics, which changed my educational trajectory as well. Through the field experience component of the course, I saw the value of classroom experience, and chose the Masters' program because it includes a year-long internship. My belief in the transformative power of the course has influenced my instructional decisions as a teacher educator and my drive to conduct research in this context. Furthermore, my interests in researching the experiences of pre-

[1] Mediated field experience methods courses are "carefully designed clinical experiences" (Darling-Hamond, Hammerness, Grossman, Rust & Shulman, 2005, p. 401). These courses are rooted in K-12 schools, spending a significant portion of the course in schools either observing or working with students.

service middle grades mathematics and sciences teachers are rooted in my own experiences of learning to teach.

To me, the most compelling element of the course is the freedom to choose the mathematical content for the after-school enrichment program. Each week we would prepare lessons for a small group of students at a local middle school. As a pre-service teacher, the inspiration for my initial lesson plans were my own mathematical interests, but as I worked with my students, I tried to tailor the lessons to their academic needs and requests. For my first lesson, I planned an activity where students created tessellations using patterns blocks and we explored the interior angles of polygons and the properties of tessellations. I worked with two 7^{th} grade girls who were shy but amenable. The students encountered some difficulties with the task I had planned, and we did not finish it in our first session. The next two sessions I followed up with additional discussions of polygons, angles, and tessellations. My purpose for following-up was twofold: tessellations are something that I think are mathematically rich and interesting; and I did not want to abandon the task just because the students had difficulty with it. Once I finished my tessellations agenda, I asked my two students what they wanted to work on together and they requested fractions. For the remaining lessons I focused on supporting students in building their procedural fluency with and conceptual understanding of fractions. I created my own manipulatives and attempted to create tasks that were cognitively demanding, rather than rote practice with fraction rules. The autonomy to select and create my own mathematical activities enabled me to take ownership of the instructional space and build my confidence in enacting student-centered teaching practices. Working with my two students each week, I gained insight into how seventh graders think about mathematics. Through my experiences in the methods course, I began to fall in love with the idea of teaching middle grades mathematics.

After completing my BS in Mathematics, I enrolled in the Masters' program, requesting a middle grades placement. My year-long mentor provided me with the same supports and freedoms that I valued in the methods course. My mentor was not ruled by the district curriculum guide, instead she made decisions about the pacing and ordering of content based on her content expertise and knowledge of her students. She did not require me to implement her lessons or activities but encouraged me test out the tasks and methods presented in university coursework. My mentor always provided honest feedback, and when lessons went awry, she was there to bring things back to order. After my internship year, I was hired at the same school, where my mentor and I worked alongside each other as colleagues for many years. She treated me as an equal and a professional from the beginning of our relationship, thus my transition from intern to teacher was a smooth one. I greatly appreciated that the school where I taught respected my instructional decisions about content and pedagogy.

In 2013, I became a mentor for teacher candidates from my alma mater. As a mentor, I tried to recreate my experiences of learning to teach for my interns. I supported their ideas and creativity, allowing them to curate their own lesson materials. I encouraged my interns to experiment, while providing them with the guidance and support that they needed to learn and grow. I enjoyed my role as a mentor: reviewing lesson plans, providing feedback, encouraging innovation, helping novices improve their noticing, modeling practices, and sharing instructional resources. My enjoyment in this role prompted me to pursue a degree in teacher education.

In 2016, I began my doctoral work at the same university. One of my graduate assistantships was to serve as a university supervisor of teacher candidates. Before I became a university supervisor, I did not fully appreciate that not every novice teacher is permitted to experiment in the classroom. Other districts, schools, and mentors were more dedicated to the provided curriculum than I or my mentor had been. The candidates I supervised struggled to balance the expectations of the university with the constraints of their placement. Many candidates did not have the opportunity to create lessons around self-selected materials.

Returning to the middle grades mathematics course as an instructor, I desired to maintain the freedoms around lesson planning and help pre-service teachers take ownership of the curriculum. I recognize the after-school program embedded in the methods course as a unique space where pre-service teachers have fewer constraints than they will encounter in their student-teacher placements. For me as a student, the after-school program was a place to explore and create mathematical tasks, which helped me to develop my epistemology and personal pedagogy as a mathematics educator. I strive to provide pre-service teachers with the same experience and benefits. As a practitioner-researcher, I wondered how the pre-service teachers experienced the course. Specifically, I was curious about how the autonomy around task selection and lesson planning impacted their understanding of ambitious teaching practices, most importantly engaging students in high cognitive demand tasks.

Research Questions

The mediated field-experience mathematics methods course provides a unique context to investigate PSTs' development of teaching practices. There is a shared context, as all PSTs deliver their instruction in the same after-school enrichment program at Bumblebee Middle School (a pseudonym). As the course instructor, I observe PSTs during enactment. PSTs' autonomy to select and implement mathematical tasks of their choosing, free of traditional institutional constraints, provides a rare opportunity to examine novice teachers' decision making as they select tasks and prepare lesson plans. The goals of this research are to understand what informs PSTs' selection of mathematical tasks and to use that knowledge to inform teacher education at the course level and beyond. Given these goals, my research questions are:

1. What types of tasks do PSTs select for the after-school enrichment program?
2. What does PSTs' task selection and lesson planning reveal about their instructional aims and their personal and pedagogical resources?

Personal and pedagogical resources include everything from PSTs' access to physical instructional materials to their beliefs about teaching and conversations with peers (Gueudet & Poisard, 2019; Gueudet & Trouche, 2009). The conceptual framework in Chapter 2 provides additional clarification as to what I mean by PSTs' resources.

Significance and Contributions

The middle grades mathematics methods course provides a unique context for examining the curriculum use of pre-service middle grades mathematics and science teachers. PSTs' autonomy to choose their own mathematical tasks for instruction contrasts with research on both pre-service and in-service teachers' curriculum use that is primarily focused on how teachers interact with predetermined curriculum (e.g., Lloyd 2008; McDuffie et al., 2018; Sherin & Drake, 2009). In other studies, when PSTs can choose their own lesson materials, it is often part of a lesson planning assignment within the methods course, and the resulting product is not enacted with K-12 students (e.g., Amador, 2019; Mathis, 2019). Furthermore, most research on curriculum use in mathematics education tends to investigate the practices of pre-service and in-service elementary school teachers (Behm & Lloyd, 2009; Morris & Hiebert, 2017; Remillard & Kim, 2017). When the curricular decisions of secondary PSTs are examined, it is often within the context of the full-time student-teaching practicum (e.g., Boroko et al., 2000; Van Zoest & Bohl, 2002). Thus, this research extends current research by investigating the curriculum use of middle grades PSTs, who are engaged in early field experiences, including the selection of mathematical tasks for instruction. Furthermore, given that research over the past 30 years demonstrates that the types of mathematical tasks students engage with heavily influences their learning opportunities (Stein et al., 2000; Tekkumru-Kisa et al., 2020), examining the types of tasks that PSTs select and why is linked to equitable mathematics instruction.

In addition to extending research on the curriculum use of middle grades PSTs, this research seeks to extend current knowledge of middle grades certification programs. The Association for Middle Level Education (AMLE) advocates for specialized licensure programs for middle grades teachers (AMLE, 2019). According to AMLE, middle grades teachers must be equipped to “select, design, evaluate, and modify curriculum in ways that capitalize on the diverse learning needs of all young adolescents” (2012, p.8). AMLE argues that middle level PSTs should have opportunities to learn about middle level curriculum through formal study and working with the curriculum in field experiences. The course context under investigation provides these opportunities for middle grades mathematics and science PSTs. Hence, this study examines an essential program element of middle level teacher preparation programs, as identified by AMLE. Examining middle grades PSTs' curriculum use is especially important because currently, there is a lack of studies on the lesson planning decision-making of middle grades mathematics PSTs.

In addition to contributing to theory, examining how the methods course influences PSTs' developing personal and pedagogical resources can help to inform the practice of teacher educators and overall programmatic decisions. Capturing how students search for and select instructional resources in a mediated field experience course can inform how teacher educators facilitate the field experience portion of the course. The method course under investigation has historically allowed PSTs to select their own mathematical tasks for their lessons in the after-school enrichment program. A possible outcome of this study is insight into how autonomy in selecting and modifying tasks impacts PSTs' opportunities to implement tasks of high cognitive demand. This insight can inform how much autonomy to provide PSTs in future iterations of the course.

At the programmatic level, this research could provide evidence for the impact of early field experiences. Not only is this study situated within a methods course with an early field experience component, but several of the study participants also completed early field experiences within courses that align to the UTeach model (University of Texas at Austin, 2021). PSTs' participation in additional early field experiences may influence their task selection and the different personal and pedagogical resources available (Sawyer & Myers, 2016). Understanding how these different early field experiences connect to and influence one another can help to structure course sequences to support PSTs' understanding of how to create and sustain learning opportunities for middle grades students. Furthermore, this research can provide insights into overall programmatic impact because it is complementary to the longitudinal study of the Middle Grades Mathematics and Science program under the direction of Dr. Dan Levin, Dr. Andy Elby, and Dr. Janet Walkoe. As part of the longitudinal study, PSTs' perceptions of mathematical tasks and procedures for lesson planning can be tracked from the mathematics methods course through the internship year and into the first year (or two) of teaching. Overall, this study has significant theoretical and practical implications, and through my current role and experience I am uniquely positioned to conduct this research.

Chapter 2: Review of the Literature and Conceptual Framework

Given that the context of this research study is a methods course, I begin by summarizing current literature on teacher education. I provide a synopsis of practice-based teacher education before detailing the literature on core practices, arguing that mathematical task selection and lesson planning are core practices of teaching. I then present an overview of curriculum use research, with an emphasis on mathematics task selection and the lesson planning practices of PSTs. Remillard's (2005) Curriculum Use Framework is discussed in-depth in the Conceptual Framework portion of this chapter.

Practice-Based Teacher Education

For more than a decade there has been a push for more practice-based teacher education (Darling-Hammond et al., 2005). Ball and Cohen (2005) argue that a practice-based curriculum "situated in the sorts of practice that reformers wish to encourage" (p. 6) could be compelling for teachers' professional learning. By embedding teacher education within the actual work of teaching, teacher preparation programs aim to close the gap between theory and practice (Darling-Hammond et al., 2005). A practice-based approach to teacher learning requires anchoring professional discussion in purposefully selected artifacts and authentic tasks involved in the work of teaching (Ball & Cohen, 2005; Ball & Forzani, 2009). Research around practice-based teacher education and professional learning captures specific pedagogies that support the unpacking and taking up of what is referred to as *core practices* of teaching.

Core practices, also known as high-leverage practices, are "a way to support teachers and teacher educators to integrate work on developing skills with work on developing the knowledge and judgment required to put those skills to use when working with students" (Grossman et al., 2018, p. 4). According to Grossman, Hammerness, and McDonald (2009), core practices of teaching are defined as:

- Practices that occur with high frequency in teaching;
- Practices that novices can enact in classrooms across different curricula or instructional approaches;
- Practices that novices can actually begin to master;
- Practices that allow novices to learn more about students and about teaching;
- Practices that preserve the integrity and complexity of teaching; and
- Practices that are research-based and have the potential to improve student achievement.

(Grossman, Hammerness & McDonald, 2009, p. 277)

Identifying the core practices of teaching is the first step to building a practice-focused curriculum (Ball & Forzani, 2009). Next, these practices must be decomposed into individual parts, or "moves", that can be developed through targeted instruction. After identifying the components of core practices, teacher educators need to prepare opportunities for the teaching and learning of the practices. Grossman, Compton, and colleagues (2009) present a framework for professional practice which is organized around the representation, decomposition, and approximation of practices. McDonald, Kazemi, and Kavanagh's (2013) cycle for learning core practices builds on the work of Grossman and others by adding the additional pedagogical categories of enactment and investigations of practice, each of which I will describe in more detail.

Representations of practice are examples of the practice in action. In the context of methods courses, representations of practice include videos or observations of classroom instruction, transcripts, or case studies, as well as modeling by the teacher educator. Each representation of practice has its own affordances and limitations

(Grossman, Compton et al., 2009). Thus, it is critical to debrief the representations of practice, so that teacher educators can be explicit about their pedagogical decisions and the function of the representation under examination. Debriefing the representation is different than decomposing the practice (*Decomposition)*, which involves breaking down a practice into smaller, distinguishable components (Grossman, Compton, et al., 2009), which can then be enacted or investigated.

Approximations of practice are an opportunity to enact practices or components of large practices in a controlled setting. Approximations of practice, by definition, are incomplete as compared to the practice of classroom teaching. The benefit of approximations is the allowance for enactment and experimentation “with more support and feedback than actual practice in the field allows” (Grossman, Compton, et al., 2009, p. 2076). In addition, approximations lower the difficulty of the task, which enables learners to focus on the essential elements of the practice, such as posing open-ended questions. Examples of approximations in methods courses include lesson rehearsals and role playing, both which involved PSTs posing as K-12 students.

Following approximations of practice, or perhaps enacting the practice in an authentic setting, PSTs should be engaged in reflection around the practice. These *investigations* of practice (Grossman, Hammerness & McDonald, 2009) include in-person debriefs, or video and audio analysis of PSTs’ approximations or enactments of teaching. Opportunities for reflection are critical for synthesizing learning opportunities.

The Practices of Task Selection and Lesson Planning

The process of selecting a mathematical task and preparing a lesson to support students' cognitive engagement with the task is a core practice of mathematics teaching. As per the criteria of core practices presented earlier in this section, lesson planning is a practice that occurs in high frequency, occurs across content and contexts, is a practice that novices can begin to master, engages novices in learning more about students, preserves complexity, and is known to impact student learning. Furthermore, effective lesson planning "sets the stage for a variety of other classroom practices related to teaching and learning," another hallmark of core practices (Grossman, Hammerness & McDonald, 2009, p. 279).

Lesson planning has been identified by both Teaching Works (2021) and AMLE (2012) as an essential component of teaching that novice teachers can begin to master. According to Teaching Works' (2021) collection of *high-leverage practices*, novice teachers should be equipped to design single lessons and sequences of lessons. Teachers should design lessons that provide opportunities for student inquiry while attending to the practice and mastery of concepts and skills (Teaching Works, 2019). In addition, teachers should be equipped to intentionally organize a collection of lessons around a central focus.

AMLE Standard 2: Middle level Curriculum indicates that middle level teacher candidates are expected to "to design, implement, and evaluate challenging, developmentally responsive curriculum that results in meaningful learning outcomes" (AMLE, 2012, p. 5). Specifically, teacher candidates are to leverage their knowledge of content, standards and students in preparing lessons for young adolescents. Middle level teacher candidates are to demonstrate their understanding of appropriate curriculum by creating learning experiences that are relevant and accessible to their students.

Mediated Field Experiences in Practice Based Teacher Education

Mediated field experience methods courses are apt for the teaching and learning of core instructional practices, such as lesson planning. These methods courses support a philosophy of practice-based teacher education by providing carefully designed clinical experiences early in the program (Darling-Hammond et al., 2005). The "mediated" aspect of the course provides consistency and coherence that is often lacking in more traditional field placements (Darling-Hammond et al., 2005). For example, in the methods course that serves as the context for this study, PSTs are provided with the same opportunities to teach and are held to the same expectations, by me, the course instructor. This consistency of experience is highlighted in contrast to PSTs' future teaching internships, where differences in districts, schools, and mentor teachers result in different opportunities for PSTs to engage in various aspects of teaching. The "laboratory-like" setting of the after-school enrichment program allows for me, the teacher educator, to provide targeted feedback on PSTs' early efforts of enacting core practices, "which can help them hone their practice before entering the more authentic, but also more complex, setting of the K-12 classroom" (Grossman, Hammerness & McDonald, 2009, p. 284). In addition, the organization of the methods course provides multiple opportunities for PSTs to enact, reflect, and refine their practices (Grossman, Hammerness & McDonald, 2009). For additional details on how the core practice of task selection and lesson planning is addressed in the methods course, see Chapter Four.

Curriculum Use

I employ the lens of curriculum use to investigate PSTs' selection of tasks and lesson planning processes, within the context of the mediated field experience methods course. Remillard (2005) defines "curriculum use" as "how individual teachers interact with, draw on, refer to, and are influenced by material resources designed to guide instruction" (p. 212). Even though this study is not examining PSTs' implementation of a formal curriculum[2], the lesson planning processes under investigation are aspects of curriculum use, thus Remillard's frameworks are applicable. In her review of research on teachers' use of mathematics curriculum materials, Remillard (2005) proposes four conceptions of curriculum use: following or subverting the text; drawing on the text; interpreting the text; and participating with the text. Each of the four conceptions are grounded in different assumptions about teaching, curriculum, and the interactions between reader and text. For the purposes of this literature review, I will describe the conception of *curriculum use as participation with the text* in more detail, as this is my perspective.

Curriculum use as participation with the text assumes that teachers and curriculum materials are engaged in a participatory relationship, where both teacher and materials are changed through their interactions (Remillard, 2005). Research situated in this perspective considers not only how teachers perceive, interpret, and utilize curriculum resources (Brown, 2009), but also how teachers are impacted by their use of these resources (Remillard, 2005). Remillard's (2005) framework of components of the

[2] "Formal curriculum" is defined by Remillard (2005) as the goals and activities established by textbooks and school districts.

teacher-curriculum relationship encompasses the features of both teachers and curriculum materials that influence their relationship. For example, teachers' pedagogical content knowledge, beliefs, identity, and perceptions of students influence how they read and interpret curriculum resources. In turn, the structures and goals of the curriculum, as well as its voice and appearance also influence how teachers interact with curriculum resources.

In considering how teachers interact with, or participate with, curriculum resources, Brown (2009) describes three ways that teachers may interact with the curriculum: offloading, adapting, and improvising. When teachers rely heavily on the curriculum materials, such as reading or presenting information directly from the teachers' guide, they are *offloading* their agency onto the materials. A teacher's interactions with the curriculum would be described as *adapting* when the teacher uses curriculum materials to guide and structure the lesson or when altering the materials. *Improvisational* curriculum use occurs when the teacher, rather than the curriculum materials, holds primary agency. For example, improvisation occurs when teachers leverage students' thinking to initiate class discussions. According to Brown (2009), each of the three types of interactions (offloading, adapting, improvising) can occur within a single lesson, as the teacher uses the materials to achieve their instructional goals.

While the types of interactions between teacher and curriculum may fluctuate throughout a single lesson, Sherin and Drake (2009) found teachers' curriculum use strategies to be consistent throughout the school year. Sherin and Drake (2009) sought to identify the strategies that in-service elementary teachers use as they work with a reform-based math curriculum for the first time. Teachers' interactions with the curriculum were coded as read, evaluate, or adapt and whether these actions occurred before, during, or after the lesson. For the duration of the study (a year or more, depending on the participants), teachers were consistent in when and how they read, evaluated, and adapted the curriculum. For example, teachers who would adapt the curriculum before the lesson by omitting components of the curriculum materials would always adapt in this manner, they would not shift to modifying existing content or adding new content. Although the approaches employed varied by teacher, the authors conclude that "teachers' use of reform-based materials, even in their first year of use, is not haphazard" (Sherin & Drake, 2009, p. 490). That is, teachers purposefully engaged in these strategies to achieve their instructional goals.

Unlike the findings of Sherin and Drake (2009), Nicol and Crespo (2006) did not find PSTs to be consist in their curriculum use strategies. Nicol and Crespo (2006) investigated the curriculum use of four elementary PSTs during their methods coursework and subsequent 13-week practicum. PSTs' interactions with curriculum materials shifted as they transitioned from developing mathematical tasks for coursework to preparing and delivering lessons to students. During coursework, two of the PSTs creatively adapted existing tasks to enhance mathematical complexity and made their own tasks. During their practicums, these same two PSTs were more reliant on the curriculum, adding on to the provided lessons rather than altering materials or creating their own. Another PST in the study shifted from an expressed desire for adherence to the textbook to considering the text as one of many resources for teaching. Only one of the four PSTs remained consistent in their curriculum use across contexts, which was characterized as adhering to the textbook. When asked about his reliance on the curriculum materials (also known as offloading), the PST stated that it was a matter of efficiency. All four PSTs identified elements of their practicum context that influenced their lesson planning decisions. The shift from theoretical lesson planning, where lesson plans are prepared for coursework but never enacted, to consistently preparing and enacting lessons with students highlights how classroom context may mediate PSTs' decisions around what and how to teach (Nicol & Crespo, 2006).

Mathematical Tasks and Lesson Planning

This dissertation study investigates a specific aspect of curriculum use: the selection of mathematical tasks. Stein, Grover, and Henningsen (1996) define a mathematical task as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (p. 460). A task may be a single problem or prompt, or a collection of problems and prompts. Task selection is a complex (Borko et al., 2000) and critical step of lesson planning. After choosing a task, teachers make instructional decisions, both in lesson planning and during implementation, that alter the selected task in ways that impact student learning (e.g., Cobb et al., 2018; Stein et al., 2007). Hence, the tasks that teachers select, the reasons for the selection, and the ways in which teachers alter tasks are important to investigate (Pimm, 2009).

Cognitive Demand

NCTM (2014) states that students' opportunities to engage in high-level thinking are contingent on teachers selecting and implementing mathematical tasks that promote reasoning and problem solving. Stein, Smith and colleagues (Smith & Stein, 1998; Stein et al., 1996) developed a taxonomy that classifies tasks on a spectrum of low- to high-level cognitive demand. The four categories of cognitive demand are: Memorization, Procedures without connections, Procedures with connections, and Doing math (Table 1). Low-level tasks (Memorization and Procedures without connections) rely on applying memorized facts or procedures, requiring little understanding of the underlying mathematics concepts. In contrast, high-level tasks (Procedures with connections and Doing math) provide for multiple entry points and solution paths, requiring students to engage in meaningful inquiry and problem solving. The levels of cognitive demand are not a hierarchy, rather they represent different goals for student learning (Stein et al., 2000). For example, if the goal is to increase students' procedural fluency, then a task that focuses on procedures without connections may be appropriate (Stein et al., 2000). However, "focusing exclusively on [lower cognitive demand] tasks can lead to a limited understanding of what mathematics is and how one does it" (Stein, Smith, Henningsen & Silver, 2000, p. 15). Stein and colleagues' extensive research around mathematical tasks found that student learning gains were greatest in classrooms where the selected tasks consistently encouraged high-level student thinking and reasoning (Stein et al., 2000). Hence, teachers' ability to identify, select, and implement tasks of high cognitive demand is critical to students' learning opportunities.

Table 1.
Levels of Cognitive Demand

Level of Cognitive Demand	Description
Memorization	• Involve reproducing previously learned facts, rules, or definitions • Are not ambiguous • Involve the exact reproduction of previously seen material • Do not connect to the concepts of meaning that underly the facts, rules, or definitions

Procedures without connections	• Little ambiguity exists about what needs to be done and how to do it • Do not connect to the concepts that underlie the procedures used • Require no explanation or explanations focus on describing the procedure that was used
Procedures with connections	• Suggest explicitly or implicitly pathways to follow that serve to connect procedures with the underlying concepts and mathematical ideas • Students must engage with the conceptual ideas that underlie the procedures in order to complete the task • Usually include multiple representations
Doing Math	• Require complex and nonalgorithmic thinking • Do not suggest a pathway or procedure • Require students to explore and understand the nature of mathematical concepts, processes, or relationships

Adapted from Smith & Stein (1998, p. 348).

Teachers' selections of tasks and other instructional materials determine the learning opportunities for students (Tekkumru et al., 2020). Thus, it is important to understand what criteria teachers use when selecting tasks and what groups of students have access to high cognitive demand tasks (Cobb et al., 2018; Tekkumru et al., 2020). However, even after selecting a higher cognitive demand task, teachers' instructional decisions during the planning or implementation stages may alter the cognitive demand of the task, potentially lowering the cognitive demand (Stein et al., 1996; Henningsen & Stein, 1997). Stein, Grover, and Henningsen (1996) found that attending to students' prior knowledge and interests were critical for engaging students in high-level tasks and maintaining the cognitive demand. If students have difficulty starting the task or lose motivation while working, teachers are more likely to intervene and routinize the task (Cobb et al., 2018; Stein et al., 1996). Thus, teachers must be equipped to identify appropriate high-level tasks for their students and know how to modify tasks to make them appealing to their students. Teachers must scaffold tasks cautiously – balancing support and encouragement without hijacking student thinking (NCTM, 2014; Henningsen & Stein, 1997).

PSTs' Task Selection and Lesson Planning

Prior research on task selection and lesson planning processes of PSTs demonstrates that PSTs gather tasks from a variety of sources, apply varying criteria when selecting online resources, and are influenced by the field experiences within the teacher preparation program. PSTs source tasks and lesson plans from their university coursework, mentor teachers, provided instructional materials, and websites (e.g., Behm & Lloyd, 2009). In their study of a PST during her culminating field experience, Borko and colleagues (2000) observed a secondary mathematics PST implementing activities and tasks from her mathematics methods coursework. The methods course provided a collection of tasks compatible with a vision of student-centered mathematics instruction. The PST said she "wanted to see how [the tasks] work in a high school classroom as opposed to a math methods classroom" (Borko et al., 2000, p. 199). The authors argue that the PST's successful transference of the tasks from university coursework to classroom practice is due to the compatibility between the methods course and the field placement.

An increasingly popular source for lesson planning resources is the internet. In Sawyer and colleagues' (2019) study of elementary PSTs' selection of resources when lesson planning, the most popular websites were Pinterest, Teachers Pay Teachers, and YouTube. PSTs' justifications for their selections included whether the resource addressed certain topics and incorporated student interests, as well as their personal experiences as learners. Elementary PSTs' search and selection of lesson resources mirrors the behaviors of practicing elementary teachers. A survey of 601 in-service elementary teachers found the most popular websites for mathematics activities to be Teachers Pay Teachers, NCTM, Pinterest, YouTube, and Education.com (Shapiro et al., 2019). Teachers' criteria when selecting online activities included alignment to standards, perceived student engagement, and level of difficulty. Both studies document that the majority of teachers who use online resources are relying on "not-so-trustworthy" websites (Shapiro et al., 2019). Given that teachers are selecting tasks from untrustworthy websites, it is unlikely that the tasks meet the criteria established by NCTM (2014).

Conceptual Framework

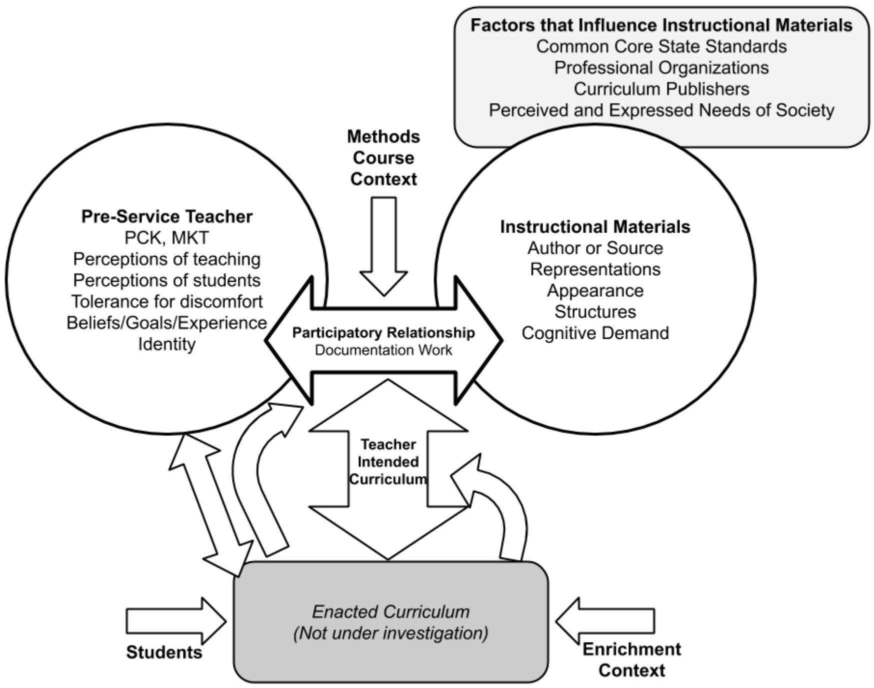
This study relies on Remillard's (2005) framework of the teacher-curriculum relationship to capture pre-service teachers' (PSTs') interactions with curriculum materials as they prepare lessons for the after-school enrichment program. In this section, I introduce my adapted framework which situates the components of Remillard's (2005) framework of the teacher-curriculum relationship within the context of this study, and incorporates elements of Remillard and Heck's (2014) model of the curriculum policy, design, and enactment system. As I discuss each component of the framework in more detail, I interweave the language of the documentational approach to didactics (Gueudet & Trouche, 2009). The participatory perspective and documental approach overlap conceptually, with the documentational approach (Gueudet & Trouche, 2009) providing more nuance to what constitutes as the participatory relationship between teacher and instructional materials (Remillard, 2019). The methods course and lesson planning assignments will be described in more detail in Chapters 3 and 4.

Operational Curriculum Framework

Remillard's (2005) curriculum use framework resulted from a review of research on teachers' use of mathematics curriculum materials. The framework encompasses the perspective of curriculum use as a participatory relationship between teacher and curriculum. For this study, I have modified Remillard's (2005) curriculum use framework to incorporate the contextual features of the methods course and elements from Remillard and Heck's (2014) model of the curriculum policy, design, and enactment system. Remillard's (2005) curriculum use framework looks at the interactions between teacher and curriculum, while Remillard and Heck's (2014) model encompasses the larger educational system and its influences on curriculum, including the participatory relationship between teacher and curriculum. The participatory relationship between teacher and curriculum is an aspect of the operational curriculum, which includes selection of instructional materials, the teacher intended (planned) curriculum and the enacted curriculum (Remillard & Heck, 2014). Thus, I have named my adapted framework the Operational Curriculum Framework. The four major components of the framework are the pre-service teacher, the instructional materials, the participatory relationship between them, and the resulting planned and enacted curriculum (Figure 1).

Figure 1.

Participatory Curriculum Use Framework Situated within the Context of this Study



Note. Adapted from Remillard (2005) and Remillard & Heck (2014).

The pre-service teacher

Like Remillard's (2005) original framework, the pre-service teacher component of the framework encompasses the intellectual resources, perspectives, and epistemologies that inform and influence how the teacher interacts with the instructional materials. For example, teachers' subject matter knowledge, or mathematical knowledge for teaching (MKT; Ball, Thames & Phelps, 2008) is one of the resources identified by Remillard (2005) that impacts how teachers understand, select, and implement instructional materials. Considering "the individual resources and perspectives of teachers helps to explain, in part, the differences seen across teachers in curriculum use, especially when they are working with the same curriculum" (Remillard, 2005, p. 229).

Gueudet and Trouche (2009) refer to these teacher resources as “operational invariants” – teacher beliefs and implicit knowledge that “are both driving forces and outcomes of the teacher’s activity” (p. 205). An example of a teacher’s operational invariant is "students' mistakes help me to design and modify my teaching resources" (Gueudet & Poisard, 2019, p. 73). Operational invariants can be individualistic or shared among teachers (Gueudet & Poisard, 2019). Operational invariants are inferred through observations of teachers as they engage in the same type of activities, such as lesson planning, across contexts.

As indicated by the arrows in Figure 1, participating with and enacting curriculum can impact teachers’ intellectual resources, perspectives, and epistemologies – all of which are encompassed by Gueudet and Trouche’s (2009) notion of operational invariants. For example, engaging with a reform-oriented curriculum may cause a teacher to shift from a direct instruction approach to teaching to a more dialogic approach to instruction (Roth McDuffie et al., 2018). This shift in instructional approach is linked to changes in the teacher’s perspectives of teaching and learning. Each of the elements within the teacher portion of Figure 1 is defined in the analytical framework found in Chapter Three.

Instructional Materials

Remillard and Heck (2014) identify instructional materials as being separate from both the official curriculum (e.g. CCSS) and the designated curriculum (textbooks and materials selected by a school or district). The broader category of instructional materials encompasses materials used by teachers that may not be officially sanctioned, whether those materials are from an online source or created by the teacher. The instructional materials that teachers use are known to have a significant influence on "both the mathematical content and the pedagogical influence of the enacted curriculum" (Remillard & Heck, 2014, p. 713). Hence, examining what materials teachers choose to use and how they use those materials continues to be of importance.

Official Curriculum

Remillard and Heck (2014) define the official curriculum as the standards and objectives that have been officially sanctioned by a governing body, such as a state department of education. In figure 1, "Factors that influence the instructional materials" represents the key components of the official curriculum, as well as organizations that have authority and influence over the creation of curriculum materials, such as NCTM and textbook publishers. For many in-service teachers, the curriculum, supporting materials, and instructional guides used by teachers are determined by the local context. Districts and schools are usually responsible for selecting and providing curriculum materials to teachers.

However, in the context of this study, the methods course, PSTs have the autonomy to seek out and select the instructional resources they use when preparing lesson plans. Despite that autonomy, the larger context of mathematics education in America does impact the materials available to PSTs both in print and online. For example, as the course instructor, I provided access to the Connected Mathematics Project (CMP) (Lappan, Phillips, Fey & Friel, 2014) print materials and a curated collection of online resources (Appendix A). Most of these recommended resources align to the CCSS and support a vision of mathematics education that aligns with NCTM's Principles and Standards (2000) and Principles to Action (2014). At the same time, PSTs also have access to websites such as Teachers Pay Teachers and Better Lesson which host lesson plans and instructional materials that do not necessarily reflect the goals and objectives of the official curriculum. Thus, given the autonomy in selecting instructional materials for use in the enrichment program. PSTs may or may not choose materials that reflect the official curriculum.

Elements of Instructional Materials

Just as PSTs' personal and pedagogical resources influence the participatory relationship, so do the features, or properties, of the instructional materials. Figure 1 identifies several critical aspects of instructional materials: author or source, representations, appearances, structures, and cognitive demand. Much of the research on instructional materials, whether of print or digital resources, has focused on the structural elements of the materials and how those features inform teacher's use (e.g. Pepin et al., 2017). Differences in the structure and organization of curriculum impact not only how one navigates the materials but also what is made available to users. For example, the linear structure of a textbook is not always replicated by web resources, which might organize lessons or tasks by the CCSS content standards or Standards for Mathematical Practice.

One defining structure of many instructional materials is the presence of supports, such as "suggestions for the teacher or actions the teacher is expected to perform and information for the teacher to read and use" (Remillard, 2005, p. 232). While it may be expected that printed teachers' guides will include supports for teachers, such as suggested questions or scaffolds for different groups of learners, online materials range in the supports provided. For example, NCTM's Figure This! website (NCTM, 2004) is a collection of tasks and solutions, but does not include additional supports for teachers. In contrast, the Mathematics Assessment Project (map.mathshell.org) provides users with not only tasks and solutions, but suggested questions, common misconceptions, and sample student work. The presence (or absence) of additional instructional supports can impact the teacher-curriculum interaction. For instance, a PST may be more likely to select tasks that are accompanied by teacher supports, because the supports reduce the cognitive load of lesson planning.

As previously noted, PSTs in this study have access to wide range of instructional materials. In addition to web resources (recommended or not) and the Connected Mathematics Project (CMP) texts, PSTs also have access to the mathematical tasks from their university coursework. PSTs are encouraged to implement the tasks from the methods and mathematics content courses with their students at Bumblebee Middle. Thus, the source, or author, of the mathematical task, whether from a website, a textbook, or a course is a critical component of the task. For example, PSTs may be influenced to choose a task authored by NCTM because they know the task aligns with the goals of the methods course and the task is accompanied by additional materials to support instruction, such as potential student responses. The cognitive demand of the task as written is also an important feature because the purpose of the enrichment session is to engage students in high cognitive demand tasks. Thus, PSTs should be assessing the cognitive demand of tasks when seeking and evaluating instructional materials and using cognitive demand as a criterion when choosing tasks.

The Participatory Relationship

The participatory relationship between teacher and materials is the focal point of the framework, and the phenomenon of interest in this study. The arrow between the Pre-serviceTeacher and Instructional Materials components in Figure 1 represents the participatory relationship, which consists of "the various ways that teachers draw on their own resources and capacities to read, make meaning of, evaluate, adopt, adapt, and replace the offerings of the curriculum" (Remillard, 2005, p. 234). Researchers have conceptualized the participatory relationship in various ways: "teaching as design" (Brown, 2009), "curriculum making" (Dempsey & O'Shea, 2019), and "documentation work" (Gueudet & Poisard, 2019; Gueudet & Trouche, 2009). Each of these different perspectives are rooted in the fact that teachers seek out, select, and modify instructional materials for various purposes. According to Gueudet and Poisard (2019), teachers' documentation work includes everything from looking at websites to selecting resources from their personal collection of lessons and materials accumulated over time. As shown in Figure 1, the participatory relationship can be influenced by the teaching context and the resulting intended and enacted curriculums. Hence, teachers' interactions with the curriculum are not static. Even as teachers continue to interact with the same instructional materials, their interactions with the materials are influenced by their evolving perceptions about teaching and students, as they enact the intended curriculum with their students and reflect on their practice.

The Intended and Enacted Curricula

The intended curriculum is the resulting product of the interactions between teacher and instructional materials, such as the lesson plan. As shown in Figure 1, the intended curriculum is a bidirectional arrow, indicating that it impacts not only what is enacted in the classroom, but can also influence the participatory relationship and how teachers draw on instructional resources. Adopting a documentational approach to didactics, the intended curriculum can also be described as the document that results from teachers' transformation of resources into documents (Remillard, 2019). This resulting document is "saturated" with evidence of teachers' operational invariants (Gueudet & Trouche, 2009, p. 205). The enacted curriculum includes how the lesson unfolds in the classroom with students. Remillard (2005) states, "the enacted curriculum is co-constructed by teachers and students in a particular context" (p. 238). Thus, as students and teachers construct learning together, the intended curriculum may change, as indicated by the arrow in the diagram pointing from enacted to intended curriculum. Finally, the enacted curriculum influences and informs teachers' future interactions with curriculum. Although an important part of the feedback cycle modeled by the diagram, the enacted curriculum is not under direct investigation in this study.

Task Selection

For the purposes of this study, I focus on one aspect of the participatory relationship: the selection of mathematical tasks. Almost thirty years of research shows that task selection remains a crucial element of mathematics education, because students' opportunities for learning are "bounded by the tasks they are assigned to work on" (Tekkumru-Kisa et al., 2020, p. 606). Complicating task selection is the enormous quantity of instructional resources made available directly to teachers via the internet

(Remillard, 2019). Given the importance of task selection and the increasing number of instructional materials available to teachers, my research explores how PSTs select and use instructional resources to create their lessons for the after-school enrichment program.

Understanding PSTs' task selection and lesson planning requires uncovering their "schemes of utilization" - the collection of goals, rules of action, operational invariants, and perceptions held by the PST (Gueudet & Trouche, 2009). A teacher's, or PST's, scheme of utilization can be inferred from sustained observation of their documentation work, such as their creation of instructional materials and lesson plans (Gueudet & Trouche, 2009). Consistent with the double arrow representing the participatory relationship (Figure 1), schemes both influence the selection and use of instructional materials and are also impacted through the use of instructional materials. This duality is illustrated by an example from Gueudet and Poisard (2019), where a teacher adapted her existing lesson materials to compliment a new digital abacus tool, which in turn reinforced her understanding of place value (MKT) and how to support students in making sense of place value. Thus, by examining the mathematical tasks that PSTs select and the lesson plans that they design around those tasks, I can infer the schemes of utilization that inform PSTs' selection and implementation of tasks – that is, I can identify their instructional aims, and the perceptions and beliefs that are encompassed their operational invariants.

Chapter 3: Research Design and Methodology

In this section I outline the design of the research study. I begin by describing my role in the research, then the programmatic context, the participants, and the methodology used. Then I describe the various data sources and the analysis procedures I used.

My role

In this study I serve as both the instructor and the researcher. I was the course instructor for the methods course, but participant recruitment and selection occurred after the conclusion of the course and submission of final grades. However, at the time of the initial interviews, seven of the participants were enrolled as students in a mathematics content course for which I was the instructor of record. Participants enrolled in this mathematics course were assured that their participation in research would not impact their grades.

Research context

This research takes place at a large mid-Atlantic public university that offers a middle grades teaching certification in mathematics and science. The middle grades mathematics and science teacher certification program provides a unique context for this work, due to the limited number of middle grades specific teacher preparation programs (Conklin, 2009). PSTs complete mathematics and science courses, some of which focused on middle school content. In accordance with state requirements and faculty reflection on what middle school teachers need to know, our PSTs also take courses in adolescent development, reading, middle school structure and philosophy, and issues of equity and diversity.

PSTs enroll in the middle grade mathematics methods course in the fall semester of their junior year. Prior to this course, PSTs may or may not have completed the mathematics content course requirements. Also, some PSTs may have participated in an early field experience course sequence based on the UTeach model (referred to as Step 1 and Step 2). Both the mathematics content courses, and the early field experience provide students with opportunities to interact with mathematical tasks of high cognitive demand.

In the fall of their senior year, PSTs enroll in the science methods course, and spend one day per week in either a mathematics or science middle grades classroom. In the spring, PSTs complete an integrated mathematics and science methods course and transition to full-time middle grades classroom presence, gradually taking over responsibility for planning, instruction, and assessment. PSTs have a choice of whether their placement is in a mathematics or science classroom.

Middle Grades Mathematics Methods Course

PSTs' coursework in the middle grades mathematics methods course and individual interviews are the data sources for this investigation. As described in the syllabus (Appendix B), the course focuses on methods of promoting middle grades student learning of mathematics, understanding the conceptual difficulties students have in moving from whole numbers to rational numbers, additive thinking to multiplicative thinking, and engaging in applications of multiplicative reasoning, including connections to geometry and measurement. There are two major threads of the course: preparing for teaching mathematics and enactment.

The first thread, preparing for teaching mathematics, encompasses mathematical content knowledge, lesson planning, and pedagogy. During the methods course, mathematical content knowledge is addressed through the completion of mathematical tasks and analyzing student work. Lesson planning is decomposed into identifying high cognitive demand mathematical tasks, anticipating student responses, planning questions to promote conceptual understanding, and differentiating instruction for diverse learners. As evidenced in the syllabus, PSTs are exposed to research-based pedagogies which promote student-centered learning and are endorsed by NCTM. These pedagogies are presented in the assigned readings and modeled by the course instructor and videos of practicing teachers.

The second thread, enactment, is addressed by the after-school program affiliated with the course. Although referred to in course documents as “tutoring” the after-school program is designed as an enrichment program, rather than a traditional tutoring service. Up to eight class sessions are relocated to a local middle school (grades 6, 7, 8) where PSTs prepare 90 minute mini lessons each week for their assigned small group of students (2 to 5 students). Middle grades students voluntarily enroll in this mathematics enrichment program. PSTs are free to choose the content of their lesson plans, provided it adheres to the Common Core State Standards for their grade-level. While there is freedom in the choice of content and selection of instructional materials, PSTs are required to use the provided lesson template (Appendix C), adapted from the Thinking Through a Lesson Protocol (Smith, Bill & Hughes, 2008). Course assignments are discussed in more detail below.

Bumblebee Middle School

This course has partnered with Bumblebee Middle School for over twenty years. Bumblebee middle school is a large public middle school close to the university campus. The school serves approximately 1,200 students in grades 6 through 8. Bumblebee

Middle is a Title 1 school and has a large population of "newcomer" students (students who have recently immigrated to the United States). As documented in the syllabus, PSTs are requested to attend Back to School Night at Bumblebee Middle School. The purpose of this request is for PSTs to familiarize themselves with the school: the physical building, the educational atmosphere, and the families served. All but one of the participants in this study were present at Back to School Night.

Participants

I recruited participants from the Fall 2018 section of the methods course. I sent a recruitment message to PSTs via the university's learning management system (LMS) at the end of the Spring 2019 semester. Ten out of 18 PSTs consented for their course materials to be analyzed, and to participate in two interviews. One of the 18 PSTs was not eligible for participation because they were a Master's student and thus were not a member of the undergraduate cohort. Table 2 displays the study participants (all names are self-selected pseudonyms), the grade-level of their students in the after-school program (Fall 2018), and their content placement for their initial internship (Fall 2019).

Table 2.
Study Participants

Pre-Service Teacher	Grade-level of tutees	Fall Internship Placement
Grace	6th	Math 8
Claire	6th	Science 8
Sara	6th	Science 7
Jake	7th	Math 8
Briley	8th	Math 8
Vincent	8th	Science 8
Jessica	8th	Math
Carson	8th (Algebra I)	Science 8
Mary-Jane	8th	Math 7
Elizabeth	8th (Algebra I)	Math 6

Note. All pseudonyms are self-selected.

Data Sources and Collection

The data sources for this study include four types of data: lesson plans, discussion board reflections, course reflections, and interviews (Table 3). Each data source is described in more detail below.

Table 3.
Alignment Between Research Questions and Data Sources

Research Question	Data Sources
What types of tasks do PSTs select for the after-school enrichment program?	Lesson Plans
What does PSTs' task selection and lesson planning reveal about their instructional aims and their personal and pedagogical resources?	Lesson Plans Reflection Posts Final Reflection Paper Interviews

Except for the interviews, the data sources were submitted as coursework. The first interview was conducted the semester following completion of the methods course. The second interview was conducted fall 2019, when PSTs were in their initial field placements. Table 4 provides a listing of the data sources and when they were submitted or collected.

Table 4.
Summary of Data Sources

Data Source	Date Submitted or collected	Context
Lesson Plans	Weekly Oct 2018 – Dec 2018	Methods Course
Discussion Board Reflections	Weekly Oct 2018 – Dec 2018	Methods Course
Course Reflection Paper	Dec 2018	Methods Course
Interview 1	May 2019	Conclusion of academic year
Interview 2	Oct 2019	Initial Field Placement

Lesson Plans

From the start of October 2018 to early December 2018, PSTs prepared weekly lesson plans for the after-school enrichment at Bumblebee Middle School, for a total of seven lessons per PST. PSTs were required to use the provided lesson plan template (Appendix C) which is adapted from the Thinking Through a Lesson Protocol (Smith, Bill & Hughes, 2008). Lesson plans included an ice-breaker, an instructional segment, and a mathematics game, totaling 90 minutes. The purpose of the ice-breaker and the game is to assist PSTs in developing rapport with their small group of students. Lesson plans were assigned a numerical grade according to the instructor developed rubric (Appendix D). The rubric assesses the lesson plan on seven dimensions: completeness, the cognitive demand of the selected task, the learning goals, attending to prior knowledge, solution methods, questioning, and engaging students in discussion. The language of the rubric is an amalgam of the rubric descriptors from the edTPA for middle childhood mathematics (SCALE, 2018) and the university's internal performance-based assessment. In addition to the rubric, PSTs also received detailed written feedback on their lesson plans before they were implemented.

Discussion Board Reflections

In lieu of an in-person debrief, following each of the seven lesson enactments, PSTs were required to post a reflection on the course's electronic discussion board. The list of prompts is provided in Appendix E. PSTs were also required to reply to at least one peer's reflection.

Course Reflection Paper

The final course assignment asked PSTs to reflect on what they learned in the course (see Appendix F). The prompts for the final paper were developed by the working group titled "Working to understand mediated field experiences and study their impact" (Swartz et al., 2018). I participated in this working group at the 2018 annual meeting of the Psychology of Mathematics Education- North America group. As a working group, we decided to pose these prompts to the PSTs in each of our mediated field experience methods courses to capture salient features of methods courses of this type and to work towards developing a continuum of how PSTs develop overtime to enact core practices.

Interviews

PSTs participated in two 30 to 45-minute semi-structured one-on-one interviews (Bogdan & Biklen, 2007), which were audio recorded. Both interviews were conducted after final grades for the methods course were submitted. I explained to the PSTs that I sought their honest responses and would not take any critique personally.

The first interview took place during May 2019. At this time, six of the PSTs were enrolled in a mathematics content course that I instructed. These PSTs were further assured that their participation and honest responses would not impact their grade in the content course. I began the interviews by asking PSTs to think back to the fall semester and describe their process for preparing their weekly lessons for their students at Bumblebee Middle. Then, I asked questions about their lesson plans and what elements of the methods course, and potentially other courses, influenced their lesson planning processes (see Appendix G for the interview protocol). Together we revisited each of their selected tasks and PSTs articulated why they selected certain tasks for their students (Gueudet & Poisard, 2019).

The second interview was conducted in October 2019, when PSTs were in their initial internship placements. This interview asked PSTs about their perceptions of coursework and preparedness to teach broadly, as well as asking them to reflect specifically on the middle grades mathematics course (see Appendix H for the interview protocol). PSTs also completed a task sort, where they were asked to categorize mathematical tasks by cognitive demand and justify their categorization (see Appendix I for tasks).

Analytic Approach

Using Remillard's framework for participatory curriculum use (Figure 1) as a guide, this study employs a combination of open-coding methods: attribute coding, descriptive coding and values coding (Saldaña, 2009). I used attribute coding to describe the elements of PSTs' lesson plans (e.g., topic addressed, cognitive demand of select task). Descriptive and values coding were used to code PSTs' characteristics and perspectives, as well as the interactions between PST and curricular resources. In the following sections, I detail the analytic processes employed for the different data sources, and how my analytic approach evolved over time.

Initial Coding of Lesson Plans

I coded PSTs' mathematical tasks and lessons plans for specific attributes. Each PST submitted seven lesson plans during the methods course. The captured attributes are: a general description of the lesson; the Common Core State Standards (National Governors Association, 2010) identified by the PST; the source or author of the mathematical task; rationale for selection of topic or task; evidence of modification; and the cognitive demand of each mathematical task. Evidence of any modifications made by the PST came from comparing the submitted task and lesson plan with the original task (if not self-created), as well as admissions from PSTs in their written reflections and interviews. Evaluating the cognitive demand of each task captures not only what types of tasks PSTs are selecting but how their modifications impacted cognitive demand, and potential trends in task demand over the course of the seven lesson plans. See Appendix J for a table containing the cognitive demand and source or author of each task, for each PST, across all seven lessons.

Initial Coding of Reflections and Interviews

I began informal data analysis during data collection. While grading the final course reflection papers, I wrote memos about course features that PSTs identified as influential to their development as teachers and learners of mathematics. In addition, I made notes during interviews when I noticed connections between PSTs' responses. Formal data analysis involved rounds of open coding (Saldaña, 2009). Interview transcripts, discussion board posts, lesson plans, and the final reflection paper were coded using descriptive and values coding using NVivo 12 software. Interview transcripts were transcribed by a transcription application and then checked for accuracy by me in NVivo.

The first round of coding employed open coding, using Remillard's (2005) framework to identify characteristics of PSTs, characteristics of the instructional materials, as well as the participatory behaviors between PST and instructional resources. Participatory behaviors included how PSTs modified tasks. Table 5 shows the general categories of code types ("look fors"), how they are defined, and examples.

Table 5.
General Coding Categories of Teacher-Curriculum Relationship

Category	Definition	Example
Teacher Characteristics		
Teacher Pedagogical Content Knowledge	PST’s understanding of how to transform their content knowledge into instruction.	“One thing I would like to improve upon is anticipating student responses.”
Mathematical Knowledge for Teaching	The mathematical knowledge needed to carry out the work of teaching mathematics.	“I fully understood these tasks because I experienced them in my ‘algebra for teachers’ math course.”
Teacher beliefs	Statements about beliefs of mathematics and teaching, that are not encompassed but the other categories (e.g. beliefs about students falls under “perceptions of students”).	“I find a lot of value in manipulatives and having students generate different representations.”
Teacher goals	PSTs’ stated goals or purpose for the lesson.	“My goal is to show them how ratios in general are two items compared to one another and seamlessly transition to using ratios in the fraction form to set up proportions.”
Teacher Pedagogical Design Capacity	PST’s ability to apply pedagogical content knowledge to design instructional contexts.	“Making a task seem as simple as possible is pretty important.”
Perception of curriculum	PST’s stated stance toward or belief about curriculum materials.	“The materials we are provided are very rigid.”
Perceptions of students	PST’s stated perceptions of the needs and capacities of students.	“They weren't like able to keep up with everything that I had planned.”
Teacher tolerance for discomfort	PST’s expressions of discomfort as related to teaching with the curriculum materials.	“I tried to look on the NCTM website to try and find something that I felt really that I knew well enough comfortably.”
Teacher identity	PST’s stated beliefs about themselves.	“I like to consider myself a creative person.”
Characteristics of Instructional Materials		
Representations of concepts	How the instructional materials depict and organize concepts including the use of diagrams, models, descriptions, and explanations.	“The provided materials are in a ‘say, see, do’ cycle.”
Look of materials	Statements about the appearance of materials.	“Thinking about like, different visual cues to give the students.”
Cognitive Demand	Level of cognitive demand of the original task and after modifications.	Assessed using the definitions of cognitive demand.
Participatory Relationship		
Types of modifications	Changes made to curriculum materials.	Changing the context of a task from hockey trading cards to Pokemon cards.

Analysis Cycle

As I continued to review the data, I engaged in an ongoing cycle of data analysis. In this section I detail my cyclical analysis process as I combed through the data to identify PSTs' instructional aims, goals, beliefs, and instructional variants as related to task selection.

Additional Codes

As I read through the data and applied the coding scheme shown in Table 5, I found the need to create more nuanced codes for the patterns I noticed. For example, when coding evidence for *pedagogical content knowledge*, I decided to create additional sub-codes for the practices and pedagogies named by PSTs. Table 6 shows the subcategories of codes that resulted from additional passes through the data sources.

Even within the sub-codes shown in Table 6 there is additional gradation. For example, I worked with fellow mathematics education doctoral students Margaret Walton and William Viviani to further analyze how PSTs were defining and operationalizing cognitive demand. I shared with Walton and Viviani the sections of the data that I coded as cognitive demand, and we engaged in several rounds of open coding to identify the different ways that PSTs defined cognitive demand and the criteria they applied when assessing cognitive demand. Once in agreement, we combined the codes into the larger themes of disposition, difficulty is related to cognitive demand, attending to student needs, and cognitive demand in relation to math understanding (Anthony et al., 2020).

Table 6.

Subcategories of Codes Implemented During Open Coding.

Code	Definition	Example
Pedagogical Content Knowledge		
Anticipating Responses	PSTs explicitly name the practice of anticipating student responses.	"Next week, I want to do better about anticipating student responses"
In-the-moment decisions	PSTs describe how they reacted to unexpected events when teaching their lessons.	"To help us move past this I had them just round the fraction to the nearest whole number so we could reach our end goal"
Questioning	PSTs explicitly identify questioning as a teaching practice.	"You could just write down so many questions that you want to ask but if half of them aren't that good then there's no power to all the questions"
Perceptions of Students		
Engagement	PSTs' comments about the importance of engagement; or how they interpreted student engagement.	"But a huge thing was, can I make this engaging?"
Student Performance	PSTs' evaluation of students' mathematical abilities; or comments related to task completion.	"They were pretty strong in their math skills."
Student Thinking	PSTs' remarks about wanting to elicit student thinking; or identifying the mathematical thinking of a specific student.	"His mathematical reasoning was different than the way I had thought…It was really cool and I hadn't thought about it before."
Perception of Instructional Materials		
Complexity	PSTs use the word complex to describe the task or mathematical content.	"With 'give an equation,' that's when I think you start to get more abstract and complex"
Cognitive Demand	How PSTs defined and described cognitive demand.	"Although there are four levels of cognitive demand, they can be spilt up into two larger categories of higher and lower level demand tasks"
Difficulty	PSTs use word likes difficult, hard, challenging, and struggle when describing the selected task or mathematical content.	"And I remember this one just wasn't very challenging. Like it just really did have much of the higher order thinking."

Identifying Instructional Aims and Operational Invariants

As part of my ongoing inductive analytic process, I wrote many analytic memos that I shared with members of my committee, fellow doctoral students, and other members of the larger mathematics education community. I began by writing narrative memos about individual PSTs. When writing about individual PSTs, I sought to weave together the patterns shown in Appendix J. For example, I wrote several memos about Claire as I attempted to understand why her initial mathematical tasks were of higher cognitive demand, then she selected several tasks of lower cognitive demand before choosing a *doing math* level task for her final lesson. As I focused on individual PSTs, I looked for patterns of thought in how they determined what mathematical content to address and which tasks to select for the after-school enrichment program. I drew several diagrams to help visualize the connections between instructional aims, the personal and pedagogical resources leveraged, and the different types of instructional materials used. PSTs' consistent behaviors and ways of thinking were markers of their operational invariants – evidence of their beliefs and "driving forces" of their selection or creation of tasks and how they designed their lesson plans (Gueudet & Trouche, 2009, p. 205). For example, in the lesson plan template, PSTs were asked to describe how they would facilitate discussion (Appendix C). PSTs' plans for eliciting student responses and the structure of discussions (e.g., going over solutions versus inviting students to make connections between different solution strategies) provided evidence of PSTs' operational invariants. As I looked across each PST's lessons, I sought evidence that these operational invariants were reinforced or challenged by each lesson planning cycle of the methods course.

During this process, it was clear that some PSTs' embodied Gueudet and Trouche's (2009) idea of operational invariants more than others. That is, PSTs' behavior across lessons was consistent, as evidenced by their tasks and lesson plans, and there was sufficient evidence to triangulate the presence of a driving belief or epistemology across the data sources (lesson plans, reflective writings, and interviews). The persistent nature of these beliefs and their influence on the resulting intended curriculum is what makes them operational invariants (Gueudet & Poisard, 2019). The PSTs who demonstrated this consistency, or invariance, throughout the methods course are the focal participants in Chapter 5 (e.g., Briley, Sara, and Jessica). The instructional aims of these PSTs became the basis for the first two set of findings in Chapter 5. Some PSTs demonstrated consistent patterns of thinking that aligned with the instructional aims and operational invariants of the focal participants, but perhaps only for a set of lessons, rather than all seven lessons. For example, Vincent shares the belief that fun tasks are tasks that are "different" than what students encounter in their regular math class, however this belief is not a driving force behind each of his lessons, thus cannot be named as an operational invariant for Vincent, because it fluctuates throughout the seven lessons.

The third instructional aim detailed in Chapter 5 is "I need to select tasks that 'fit' my students." This instructional aim was expressed by 9 of the 10 PSTs. While no one PST embodied this instructional aim in the same way as the other two main findings, the prevalence of this instructional aim across the participants cannot be ignored. Given the variety of ways that PSTs defined and assessed task fit, there is not a singular operational invariant associated with this instructional aim. Rather, PSTs' goal of selecting tasks that "fit" might capture PSTs' responsiveness to the dynamic nature of the lesson planning and enactment process, as capture by the feedback loops in the conceptual framework (Figure 1).

Chapter 4: Middle Grades Mathematics Methods

This chapter provides an overview of the middle grades mathematics methods course. While the course itself is not under investigation in this research, it serves as context to illuminate the types of activities that PSTs experienced. I begin with a general overview of the course before detailing the course elements that supported the core practice of task selection and lesson planning. I use the Cycle for Learning Core Practices (McDonald et al., 2013) to explain the pedagogies used to provide PSTs with opportunities to learn about, practice, and reflect on mathematical tasks and lesson planning. Although the course was not designed around the Cycle of Core Practices, I use the learning cycle to connect to current research in teacher education.

Overview

The middle grades mathematics methods course is the undergraduate version of the Methods 1 course described in Grosser-Clarkson (2016). The primary differences between the undergraduate and master's-level versions of the course are the length of class sessions and access to middle grades students. Considering the overlap in context, much of the following description of the course activities and pedagogies employed mirrors what is found in Grosser-Clarkson (2016).

Typically, PSTs enroll in the mathematics methods course the fall semester of their junior year. The course meets for 110 minutes, twice a week from August to December. Starting in October, one class session a week is held at Bumblebee Middle school, where PSTs work directly with small groups of middle grades students. The first five sessions of the course introduce students to mathematics education and course philosophies, including the CCSS, the Standards for Mathematical Practice, and issues of equity in mathematics education. The following four sessions introduce students to lesson planning practices, such as identifying tasks of high cognitive demand, launching tasks, and effective questioning practices. Subsequent class sessions continually revisit these topics, while incorporating additional concepts such as facilitating discourse, supports for diverse learners, and methods of assessment. Concurrently, PSTs implement course concepts through the planning and delivery of instruction to their students at Bumblebee Middle.

Cycle for Learning Core Practices

The Cycle for Learning Core Practices was developed by McDonald, Kazemi, and Kavanagh (2013) to serve as a framework for teacher educator pedagogies that support PSTs in learning to enact core practices (Grossman, Hammerness & McDonald, 2009). A cycle is an appropriate model for PST learning given that "deep understanding involves returning to central ideas and concepts again and again, so that over time [PSTs] are able to understand them more thoroughly and to appreciate their relationship to other concepts, ideas, and theories" (Darling-Hammond et al., 2005, p. 398). Thus, as PSTs move through the cycle of learning, their experiences in each part of the cycle inform and influence their subsequent experiences. The cycle is composed of four parts, each of which I will elaborate on in further detail, while connecting to the specific pedagogies employed in the methods course.

Figure 2.

Cycle for Learning Core Practices

Note. From McDonald, Kazemi, and Kavanagh (2013, p. 382).

Representations of practice

Although PSTs may enter the Cycle for Learning in any of the four quadrants, often teacher educators will begin in the top right quadrant, *Introducing and Learning About the Activity*, where PSTs are first introduced to a core practice. The teacher education pedagogies in this phase of the cycle are the *Representations of Practice* and *Decomposition of Practice* (Grossman, Compton, et al., 2009). Representations of practice "provide a common moment of teaching to discuss and unpack" (Danielson et al., 2018, p. 15). In the mathematics methods course, I attempted to employ a variety of pedagogies to engage PSTs in learning about the different practices. Specific pedagogies used in the instruction of the core practices of task selection and lesson planning were Task Comparisons and Modeling.

Task comparisons

PSTs were introduced to the concept of cognitively demanding tasks through the task sort created by Smith and Stein (1998). Prior to class, students were asked to review the definitions of the four categories of cognitive demand (Smith & Stein, 1998, p. 348). In class, we revisited the definitions and addressed questions posed by students. After highlighting the distinctions between each of the categories, PSTs then worked in groups of three to sort eight tasks into the four categories (Smith & Stein, 1998, p. 346). PSTs posted their categorizations in a chart on the dry erase board. Together as a class we worked together to rectify any conflicting categorizations before comparing our decisions to the classifications and justifications provided by Smith and Stein (1998). As we discussed the authors' intended classifications for the tasks, we acknowledged the role that students' prior knowledge plays in determining the cognitive demand of the task. Thus, the cognitive demand of a task is variable depending on the context. This task sort is an example of a decomposition of practice because a critical aspect of identifying and selecting appropriate mathematical tasks is being able to assess the cognitive demand of the task.

Modeling

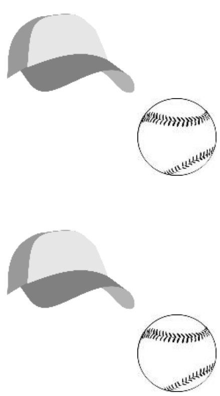
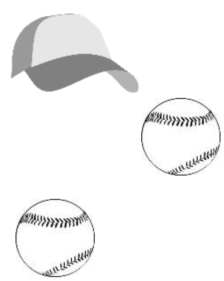
The purpose of modeling in a methods course is to "make cognitive work visible so that learners are able to see and begin to take up the thinking and decision making integral to the discipline" (McGrew et al., 2018, p. 37). The teacher educator decomposes and enacts the practice, while metacognitively marking the work to invite PSTs into the decision-making process (McGrew et al., 2018). In the mathematics methods course, modeling the enactment of rich mathematical tasks is one of the most used pedagogies (Grosser-Clarkson, 2016). Asking PSTs to engage in mathematical tasks as learners pushes PSTs to develop multiple solution methods, link those methods to student understandings, or misconceptions, of the content (Danielson et al., 2018; Grosser-Clarkson, 2016). Ultimately, “these mathematical tasks serve a context for the instructor to model ambitious teaching” (Grosser-Clarkson, 2016, p. 60). See Appendix I for a table of the mathematical tasks modeled during the course.

The teacher educator’s modeling of practices must incorporate a debrief that includes analysis of “the teacher’s choices about the content, context, student needs, and learning goals” (Danielson et al., 2018, p. 39). After PSTs completed mathematical tasks as learners, we would debrief the task by identifying the underlying mathematics of the task, the grade-level standards addressed, the Standards for Mathematical Practice employed, and the cognitive demand. Each of these components are relevant to task selection and lesson planning, for they help PSTs determine when in a learning progression a task could be used. For example, The Baseball Shop task (Figure 3) addresses the CCSS (2010) Math 8 cluster 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations. However, the task can be solved without writing and solving a system of equations in standard mathematical notation. When the more informal algebraic approaches are purposefully linked with solving the system of equations, the task can be classified as Procedures with Connections. As a class we discussed when during a sequence of lessons on systems of equations a teacher would choose to engage students in this task.

Figure 3.

The Baseball Shop Task

A baseball souvenir shop is offering the following packages for signed baseballs, hats, and bats:

Couple's Package **$46.00**	**Autograph Hunter Package** **$31.00**	**Let's Play Ball Package** **$33.00**
		

If the baseball shop sells each item individually, what is the cost of each item?

Approximations of practice

Next, PSTs enter the bottom right quadrant, *Preparing for and Rehearsing the Activity*, which may involve *approximations of practice* (Grossman, Compton, et al., 2009) or collaborative planning. Approximations support novices in the shift from thinking about what they *might* do to what they *will* do (Schutz et al., 2018). McDonald, Kazemi, and Kavanagh (2013) present examples of the learning cycle across settings. In a more authentic setting, approximations of practice include lesson planning, anticipating student responses, and developing questions to ask. Given PSTs' autonomy in preparing lessons for their students at Bumblebee Middle School, I find the context of the field experience to be more authentic than controlled. Although PSTs' work with students is part of an after-school program (and not whole class instruction), the lesson planning experience is an authentic approximation of practice because PSTs have control over the content and instructional decisions.

Preparing lesson plans

Prior to preparing their own lesson plans, PSTs read about the Thinking Through a Lesson Protocol (TTLP) (Smith et al., 2008). In class we discussed the difference between producing a lesson plan (creating a document) and engaging in the act of lesson planning (a process). The TTLP functions as a guide to lesson planning, through its modeling of questions that teachers should pose to themselves when preparing to teach. PSTs were required to use the provided lesson plan template (Appendix C) to prepare their lessons for enrichment at Bumblebee Middle. The lesson plan template was designed to follow the TTLP. The prompts for each section of the template mirror the questioning of the TTLP. The template itself serves as a decomposition of practice because it breaks down the components of a lesson, uses the language of the course, and models teacher's internal decision making (Danielson et al., 2018).

Each week, beginning in October, PSTs submitted the completed lesson plan template. PSTs received feedback on their lesson plans via in-line comments from the instructor and the lesson plan grading rubric (Appendix D). The language of the lesson plan grading rubric mirrors that of internal university performance-based assessments. In relation to the practice of task selection, the rubric evaluates PSTs on whether they have chosen a high cognitive demand task and if their proposed implementation maintains the cognitive demand. As noted in Chapter Two, the categories of cognitive demand are not a hierarchy. However, given the importance of implementing high cognitive demand tasks and the difficulty in doing so (Stein et al., 2000), I find it an essential element of the course to motivate PSTs to select and implement cognitively demanding tasks.

To assist with the selection of high cognitive demand tasks, PSTs are provided with a list of online resources (Appendix A) and access to the CCCSS aligned Connected Mathematics Program materials (Lappan et al., 2014). Prior to working with students at Bumblebee Middle School, PSTs collaborated in grade-level groups. This collaborative time is unstructured, allowing PSTs to explore the print materials and workshop lesson plan ideas with peers. The cohort under investigation also created a shared GoogleDrive folder where they uploaded their lesson plans and collaborated virtually. This shared space was created and maintained by the PSTs, independent of the course, and not monitored by me, the course instructor. In interviews, PSTs described the shared folder as a space where they could exchange ideas regarding “what worked” about their lessons. PSTs’ collaborative actions are approximations of the professional learning activities that they will engage in as classroom teachers.

Enactment

After preparing for the activity, PSTs then *Enact the Activity with Students* (bottom left quadrant of Figure 2). During this phase, PSTs enacted their weekly lesson plans with small groups (two to five) of middle grades students at Bumblebee Middle. PSTs facilitated their lessons in a common space, so that I had simultaneous access to all groups. As the teacher educator, I supported PSTs during their lesson enactment by modeling questioning practices, making real time suggestions, and providing written observation notes at the end of the session. PSTs also audio recorded one of their enrichment sessions, which they analyzed as a course assignment (see description of audio analysis assignment in Appendix B).

Investigations of practice

Finally, teacher educators engage PSTs in *Analyzing Enactment and Moving Forward* (upper left quadrant of Figure 2) using *investigations of practice* (Grossman et al., 2009). According to McDonald, Kazemi, and Kavanagh (2013), the reflective and analytic work that PSTs undertake during the cycle is “a key aspect of giving meaning to the practices that are being worked on” (p. 383). Hence, PSTs’ learning of a specific practice does not end with reflection – the cycle continues, with PSTs’ reflections informing the next phase of interactions with representations of practice. In the mathematics methods course, the primary pedagogy for engaging PSTs in reflection around task selection and lesson planning was weekly discussion board posts.

Weekly discussion board posts

Following the lesson enactment at Bumblebee Middle, PSTs engaged in debrief and reflection via online discussion board posts. PSTs submitted five reflections on their lesson enactments. The reflection prompts asked PSTs to reflect on their work with students and to set goals for the following sessions. See Appendix E for the reflection prompts. Goal setting is an important aspect of reflection because it initiates the next cycle of learning. The discussion posts also invite PSTs to investigate the practices of their peers, through reading and responding to peers' posts. As the teacher educator, I supported PSTs' reflections and investigations of practice by affirming their goals, making suggestions for how to meet their goals, and addressing questions posed.

Chapter 5: Findings

When preparing their lessons for enrichment at Bumblebee Middle School, PSTs selected tasks from a variety of sources, and of varying levels of cognitive demand. Table 7 below shows the total number of tasks by source type and level of cognitive demand. For more details about the task sources or the selections of individual PSTs, see Appendix J. As shown in Table 7, approximately one-third (24 of 73) of the mathematical tasks were self-created, which tended to be of lower cognitive demand (17 of 24). However, more than half of all selected (or created) tasks were of higher cognitive demand (39 of 73).

Table 7

Tasks by Source and Level of Cognitive Demand

	Source or Author							
	Self	Recommended (e.g. NCTM; MARS)	Math Course	Methods Course	Peer	From Instructor	Other Websites (e.g., TPT, worksheet sites)	Total
Memorization	6	1	0	0	0	0	5	12
Procedures Without Connections	11	2	0	1	0	0	8	22
Procedures With connections	7	8	4	6	4	1	6	36
Doing Math	0	2	1	0	0	0	0	3
Total	24	13	5	7	4	1	19	73

Note. Some lessons included more than one task.

In this chapter, I unpack PSTs’ task selection and organize my findings around the common instructional aims and operational invariants that informed PSTs’ task selection and lesson planning. Given that the Middle School Mathematics and Science teacher preparation program operates under a cohort model, it is not surprising to find shared beliefs or ways of thinking among the participants. For each of the three themes, I describe the shared aim or operational invariant and demonstrate how it combines with other personal and pedagogical resources to form a scheme of utilization which informs PSTs' participation with instructional resources. In Chapter 6, I will discuss how teacher educators can leverage these instructional aims and operational invariants to support PST learning.

PSTs Just Wanna Have Fun

A shared instructional aim among PSTs was for students to have fun with mathematics during the after-school enrichment sessions. For example, Jessica articulated the goal of having fun when replying to Briley's lesson reflection on the discussion board:

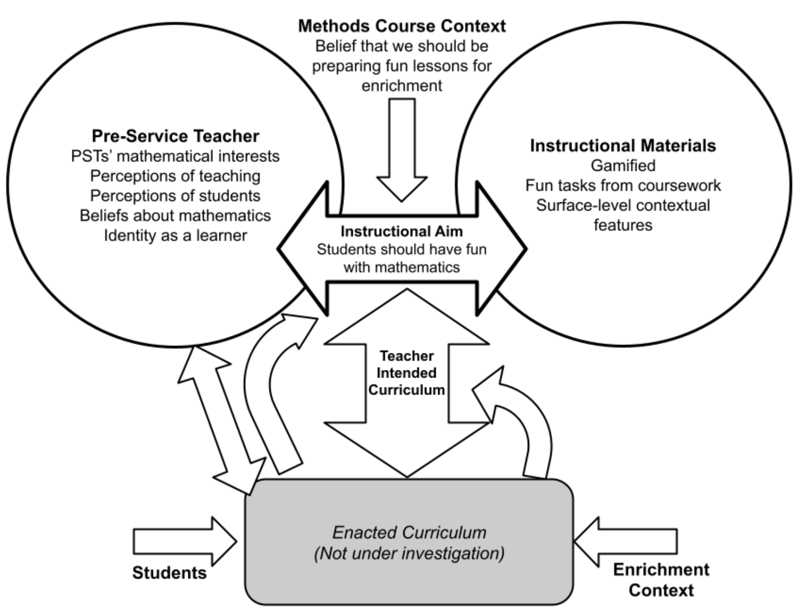
> I think it is great that your students are having fun and enjoying themselves. The sessions are so valuable because they are able to experience math in a more relaxed environment, unlike their classroom may be. I told my students earlier in the semester that the sessions were the time for them to have fun-- it was not supposed to feel like they were in class. It sounds like you are doing a great job at making sure your students are having fun and enjoying themselves while still teaching them something valuable. (Discussion Board Nov 29)

Jessica evaluates Briley's lesson as successful because Briley is ensuring that her students have fun. In addition, in her reply, Jessica shares that she believes that the purpose of the enrichment sessions is to have fun and she communicated this belief to her group of Algebra I students. Like Jessica, Mary Jane also articulated that the purpose of the enrichment sessions is to for students to have fun. Mary Jane said the task for her first enrichment session was motivated by a desire for "[students] to realize this isn't just another like math tutoring thing. But this is supposed to be fun and engaging" (Interview 1). Mary Jane saw her first task as setting the expectations for the rest of the enrichment sessions – an expectation of fun.

Since PSTs' belief about the purpose of the enrichment sessions informed their instructional aim for students to have fun during enrichment, the resources activated by this instructional aim include PSTs' beliefs about what makes a task fun and what makes mathematics fun. These beliefs are informed by PSTs' experiences as learners, perceptions of teaching, perceptions of students, and their personal mathematical interests (Figure 4). I begin by identifying the operational invariants of PSTs who focused on features of fun tasks and then discuss PSTs whose sense of fun is rooted in the mathematics.

Figure 4.

Instructional materials and resources leveraged as PSTs select and prepare fun tasks



What Makes Tasks Fun

Some PSTs demonstrated an operational invariant of "*tasks are fun when they are different from what students encounter during their regular mathematics classes*." For these PSTs, the format or features of the task serve as the vehicle for fun, independent of the mathematics embedded in the task. Gamification, task context, and student choice are all surface level features of the task that PSTs think will make the content more fun for students. By surface-level, I mean that changing any one of these features will not significantly alter the mathematical content of the task. For example, changing the context of a word problem about baseball cards to Pokémon cards to make it more relatable would be a surface-level adaptation. To further illustrate PSTs' schemes of utilization to achieve the instructional aim of having fun, I begin with a portrait of Briley, who included game design elements in all of her lessons. Then I provide examples of Vincent's lessons where surface-level features attempt to make mathematics fun. For the PSTs who foreground the fun of the task itself, the goal of selecting or creating fun tasks overshadows engaging students with the mathematics.

Gamification

For Briley, gamified tasks are fun. All of Briley's tasks included some game design elements, such as tokens, earning points, and competition. Briley's choice of game elements was independent of the mathematical content of the activity. Briley felt like her second lesson, where students matched equivalent exponent expressions and derived exponent rules (e.g. multiplying like bases), "felt too formal and classroom-like" (Discussion board post Oct 14). She wrote that she wanted to "make my [next] lesson a little more exciting and fun; I want to incorporate some kind of challenging game or competition possibly" (Discussion board post Oct 14). For her next lesson, Briley created a board game to practice solving equations:

> I am going to create a game board by placing 36 notecards in a square. Each student will begin at "start" and roll the die to go around the board. When they land on a square, they will pick up the card. First, they have to determine if the expression has one solution, no solutions, or infinite solutions. We will organize the cards into three separate piles, and when they get a card correct, they get a point and we will keep track of their scores. The space that says 'solve a one solution,' means that they get to roll the die and choose a card from the one solution pile and solve with the number they roll. If they get it correct, they get a point. The game keeps going until there are no more spaces left on the board; whoever has the most points at the end, wins. (Lesson Plan 3)

To determine whether the equations have one, none, or infinite solutions, students had to solve the equation on the card. I assessed this task as procedures without connections, because of the focus on procedural fluency and applying definitions. Students were not invited to make sense of what it means conceptually for an equation to have one, none, or infinite solutions. Furthermore, during the lesson Briley found herself having to guide students step-by-step through each of the problems. In response to students' lack of procedural fluency, her next lesson focused on solving one and two-step questions. Briley recognized that "this is a relatively straightforward skill" so she tried to make the lesson more enjoyable by embedding the repeated practice within a Jeopardy game (procedures without connections). In her post lesson reflection, Briley assessed the lesson as a success: "I think this game was really fun for them and they were excited about playing the entire time. They were upset when it was time to move on" (Discussion board post Oct 28). Thus, Briley's belief that her self-created games were fun was reinforced by students' professed enjoyment of the Jeopardy game.

In her final discussion board reflection, Briley wrote, "I think that my students learned how to have more fun with math…I think that when we do activities that are more like games, or just things they don't usually do in class, it seems less like a lesson and more like a fun activity" (Discussion board post, Nov 29). The mathematics of Briley's enrichment lessons was the same as the content that students had encountered or were encountering in their mathematics courses. What made the lessons "fun" was the format of the task, not the mathematics addressed. While the cognitive demand of Briley's tasks, as designed, fluctuated between procedures with and without connections (Appendix J), the goal of having fun overshadowed the mathematical objectives.

Briley designed all her lessons herself, with the exception of her second lesson (exponent rules). When asked why she created her own tasks rather than use or adapt existing tasks, Briley stated:

> I feel like I just like to do things by myself. Or I just like I like to think of ieas by myself and I can visualize things when I create them myself a lot better than when I look for other people's things. I guess, I like to consider myself a creative person. (Interview 1)

Thus, one of Briley's resources activated during lesson planning is her identity as a creative person. As a result, Briley is primed to create her own tasks rather than seek out and select existing tasks. When asked about her process, Briley said that she would read the CCSS and then go with the first idea that came to mind – "that's just the kind of person that I am…it's easy for me to make stuff and visualize things like that" (Interview 1). These statements imply that Briley's tasks embody her beliefs about what funs tasks are. Furthermore, these beliefs are reinforced by her perceptions of student enjoyment of her lessons.

Surface-Level Contextual Features and Student Choice

Multiple PSTs demonstrated that contextual features of tasks are what make tasks fun. PSTs were drawn to tasks that were guised in seemingly real-world contexts, had hands-on elements, or were "festive." For example, Vincent's first enrichment lesson was "grounded in a fun reality" (Interview 1). Vincent provided his 8th grade students with pamphlets advertising vacation packages and students were asked to calculate the total cost of their selected vacation, while incorporating discounts and calculating the overall sales tax for their vacation package. Upon reflection, Vincent assessed his first lesson as a success:

> So, the first day was just a lot of fun. And it also really helped me get to know them. Because if I went in with [a worksheet] they'd be like, "Mr. Taylor is the most boring person ever." But I went in with something that they really have control over. And I had fun working with them. And I think they had fun doing it as well. It's just one of the activities I wish I got when I was in middle school. So, I use my miserable middle school experience to like, flip it. It's like one of the main reasons I want to do it. (Interview 1)

Like Briley, Vincent contrasts his first task with worksheets and students' "typical" experience in mathematics classes. He also leverages his experiences as a learner. Vincent has created a task that contrasts with his middle school mathematics experience. For Vincent, an important feature of the task that made it fun was the opportunity for student choice. In this activity, students chose the different elements of the vacation package.

Vincent included the element of choice in his next lesson where students were designing their own houses. When preparing for the lesson, Vincent envisioned students building houses out of cardboard and then calculating the areas of the walls. During implementation, Vincent encountered some difficulties as students were cutting and measuring the cardboard. He did not anticipate that his hands-on activity would result in mixed numbers:

> I didn't account for the fact that when we measured each side of the box, that there would be mixed number fractions…I was not prepared to go over the operations and how we treat each fraction. This led to a large chunk of time where they were struggling working with fractions. During this time, even I lost track of what we had as our end goal. (Discussion board post, Oct 14).

Despite the devolution of the task, Vincent identified the context of the task as a successful element of the lesson: "The students definitely enjoyed the overall task of creating their own houses and seemed excited to get to work initially" (Discussion board post, Oct 14).

Both of Vincent's lessons described above have surface level real-world connections. The mathematics of each of the lessons is not dependent on the context of

the task itself, thus these tasks are not truly real-world mathematics tasks or "modeling with mathematics" (Standards for Mathematical Practices). Furthermore, these two lessons are classified as procedures without connections, because students perform the calculations but without linking procedures to the underlying concepts. Like Briley, the goal of making mathematics fun overshadowed creating, or selecting, tasks of higher cognitive demand. When reflecting on the semester, Vincent wrote, "As far as goals for mathematics as a whole, I share the goal of making it more approachable and fun for the students and hopefully creating a spark of curiosity to learn more" (Discussion board post, Nov 29). While Briley's instructional aims for having fun with mathematics may be localized to the enrichment context, Vincent's aims are more far reaching:
"I feel that supporting this curiosity is one of the most important things I can do to, and therefore is a goal I have for the next session as well as for my career in general" (Discussion board post, Nov 29). Together, these two quotes from Vincent demonstrate an operational invariant of "*fun leads to curiosity*."

What Makes Mathematics Fun

While most PSTs who communicated a belief that their enrichment lessons should be fun focused on the features of the task themselves, two PSTs were drawn to mathematical content that they found fun. Both Elizabeth and Mary Jane selected tasks that satisfied their goal of making mathematics fun, while also engaging students in tasks of higher cognitive demand (procedures with connections). Elizabeth's personal enjoyment of Algebra and her interest in patterns tasks can be interpreted as an operational invariant of "*algebra is fun*." Elizabeth chose to work with a small group of

8th grade students who were in Algebra I. Her choice of grade-level was based on her interest in the content:

> I really enjoy algebra. So, anything in the scope of algebra really draws me in… In the beginning of the semester, I would really think about like, how I would like to learn it, and how the students might respond if I said certain things, also thinking back to [an earlier field experience course], anticipating some of those ideas. (Interview 1)

Elizabeth's first lesson used a task from NCTM's Illuminations website that invited students to explore the difference of squares. When reflecting on the task, Elizabeth said,

> But I think for the difference of squares, I never knew about it…I really enjoyed the concept, because I thought about it and I was like, "I never noticed that." So, I guess part of what draws me to [tasks] is like, well, besides genuine interest, is if I've never seen it before, it's something that the students might have never seen before. (Interview 1)

As shown in the quote above, Elizabeth's sense of mathematical curiosity motivated her selection of this task. Elizabeth expressed a hope that since she found this topic interesting and enjoyable, that students will also be intrigued by the topic. Elizabeth's selected task has a cognitive demand of procedures with connections because it links together multiple representations and invites students to make and test conjectures. In contrast to Briley and Vincent's tasks, the fun of Elizabeth's task is rooted in the mathematical discovery, rather than add-ons such as game elements or choose your own adventure.

Mary Jane selected a task that she perceived to be fun because of the opportunity for discourse. Mary Jane selected the Random Rectangles task from her statistics course for middle grades teachers. The task addresses the concepts of random sampling and measures of central tendency. Students are asked to randomly select different amounts of rectangles from a collection of 100 rectangles and perform calculations. The Random Rectangles task is assessed as procedures with connections because it engages students in sense-making around random sampling and the appropriateness of the measures of central tendency in describing the data set.

Mary Jane's selection of the Random Rectangles task is motivated by a sense of familiarity with and fondness of the task. As to why she selected the task for her first lesson, Mary Jane stated that she remembered "having fun with my classmates when we did [this activity]" (Interview 1). Thus, one of Mary Jane's resources is her experience with the task as a learner. She completed the task in her statistics course, thus she "knew what was supposed to happen" (Interview 1). While Vincent created tasks that contrasted with his experiences as a learner, Mary Jane has selected a task that she enjoyed as a learner. Mary Jane said that she chose this task for her first enrichment session because she wanted "[students] to realize this isn't just another like, math tutoring thing. But this is supposed to be fun and engaging" (Interview 1). Thus, like Briley and Vincent, Mary Jane juxtaposes her selected task with students' typical experiences in their mathematics classes or with tutoring.

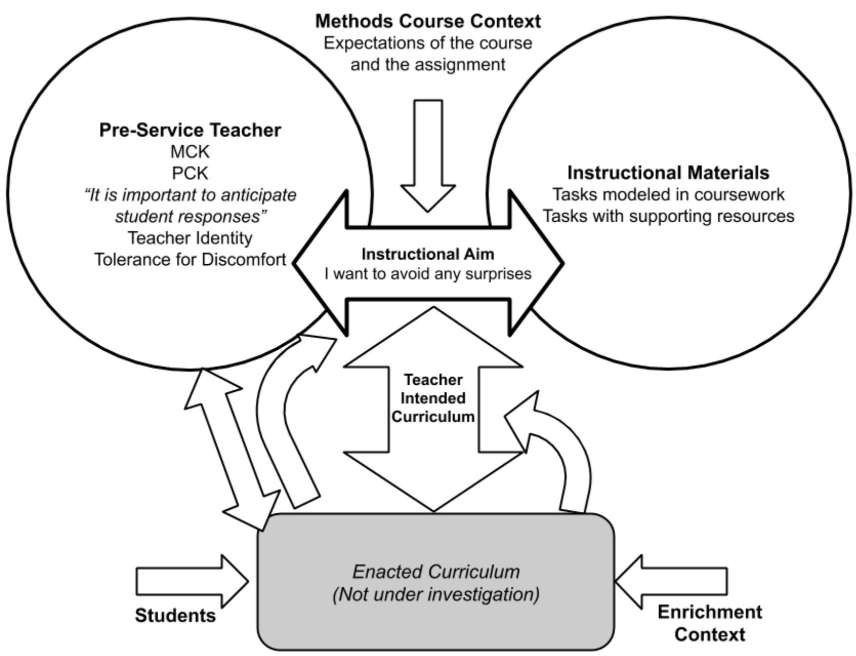
Mary Jane's assessment of what makes the Random Rectangles task fun or different from other tasks are the expectations placed on students, not the features or presentation of the task. Thus, The Random Rectangles Task stands in contrast to the tasks previously discussed because the "fun" of the task is rooted in the mathematics, not the surface-level features. As a learner, Mary Jane was engaged in discourse that supported her conceptual understanding of measures of central tendency and random sampling. Thus, Mary Jane understood a goal of the Random Rectangles task as engaging students in discourse. In her lesson plan, Mary Jane wrote that the task "emphasizes on sharing perspectives with one another." By the end of the lesson, she wants "students to be able to explain the definition of randomness in their own words while explaining why random sampling is important" (Lesson Plan 1). For Mary Jane, this task represents the types of prompts students should be working on in mathematics courses: "I think these are the kind of questions that students need more of, instead of just like practicing a worksheet with 30 repetitive problems" (Interview 1). Thus, the Random Rectangles Task is not only "fun" but is representative of Mary Jane's perception of what it means to teach mathematics with tasks.

Anticipating Student Responses to Avoid Surprises

The aim of "avoiding any surprises during the lesson" appears to be motivated by PSTs' identities as novice teachers and their tolerance for discomfort (Figure 5). For most of the PSTs, the methods course is the first time they had sustained contact with middle grades students. Furthermore, the selected tasks and prepared lesson plans must meet the course expectations, which means selecting a task of higher cognitive demand and utilizing the five discourse practices (Smith et al., 2009). I identify the operation invariant associated with this aim as "*It is important to anticipate student responses*." Across the dataset, there are 33 instances of PSTs commenting on the importance of anticipating student responses as an important component of teaching and lesson planning – there is at least one statement from every participant. While the aim of "avoiding any surprises during the lesson" was displayed by multiple PSTs, it is best articulated by Sara and Jessica. In this section, I will describe how the shared operational invariant of "It is important to anticipate student responses" resulted in Sara utilizing instructional materials from recommended websites, while Jessica drew from course materials and lessons created by her peers.

Figure 5.

Instructional materials and personal resources leveraged as PSTs seek to avoid surprises



Sara's Use of Educative Materials from Recommended Websites

Sara's instructional aims appeared to be static throughout the semester, as she continued to communicate a need to control the mathematical content of the lessons and how the lessons unfolded. Sara's process for selecting mathematical tasks was consistent throughout the semester. She only selected tasks from instructor recommended websites (e.g., NCTM, Balanced Assessment, Inside Mathematics).

Sara's Personal and Intellectual Resources

Sara's aim of "I want to avoid any surprises" is reinforced by her identity as a novice science teacher and her lack of self-efficacy in mathematics. Repeatedly during the first interview, Sara foregrounded her status as a novice mathematics teacher:

> I don't want to steer them in the wrong direction. I am not qualified to teach. So I did not want to tell them something or do something that was, you know, that their teacher might be like, "Well, where did you hear that from?"…Because like I said, I don't want to steer them in the wrong direction…I didn't want to take away from like, their actual educa- their actual public education and their teacher.

In these quotes, Sara contrasts herself with her students' "actual" mathematics teachers. She is wary of making instructional decisions that are at odds with their classroom teacher. Sara is concerned that her explanations will be detrimental to her students' learning. Despite her prior teaching experiences, Sara does not yet see herself as a "real" teacher.

One explanation for Sara's lack of self-efficacy with mathematics is her preference for science. Prior to the methods course, Sara completed the two introductory courses of the university's UTeach program. In the first course, undergraduates plan and teach a lesson in a local elementary school. In the second course, undergraduates plan and teach two lessons in a local middle school. Undergraduates have a choice of preparing either math or science lessons, and they prepare and teach the lessons in groups of two or three. Additionally, the lesson tasks are preselected by the course instructors. When given the option to teach science or mathematics, Sara chose science:

> At the time I was secondary science and I know in that you can either have math or science, so I was doing more science-based, which I am more comfortable with because I... like I like science and I retain it a lot easier. (Interview 1)

Across both interviews Sara articulates confidence in her science content knowledge and an unease about mathematics:

> [S]cience has always been a thing that I've always understood and I've kind of always really loved. And like math, it kind of it was kind of difficult for me. I mean, it still is sometimes, especially if it's something I haven't done in a while. (Interview 2)

However, in the methods course, Sara was obligated to plan her enrichment lessons around mathematics content. At Bumblebee Middle, Sara chose to work with 6th grade students because she felt most comfortable with that mathematics content. Even still, Sara's written reflections capture her pervasive lack of confidence in her pedagogical content knowledge: "I wasn't sure if I was explaining it the correct way" (Discussion board post, Oct 7); "Sometimes I worry about making sure I have the right questions lined up for the students, or to make sure I facilitate a productive discussion." (Discussion Board Post, Oct 21).

Given her preference for science and her lack of confidence with teaching mathematics, Sara was thankful that she was able to choose the mathematical tasks for her enrichment lessons:

> I did like that [for the enrichment sessions] I could choose like, if I chose whatever the concepts were that I wanted to talk about, I could teach that - I could choose the actual lesson plan that went with it. And that's what I really liked about the NCTM website. Like, I think I'm the only person who actually bought the membership… So I did, I did like that I had the control over what the actual lesson plan was... (Interview 1)

The recommended web resources, specifically from NCTM, appear to provide Sara with a sense of comfort when preparing tasks each week. Although she is struggling with a lack of self-efficacy in mathematics, these tasks have been created and curated by more experienced mathematics educators. Thus, these tasks possess an authority that she does not yet assign to herself. In addition, the NCTM resources are educative, and are aligned with the goals of the methods course and the lesson plan expectations.

During the initial interview, Sara stated that she appreciated the autonomy of the enrichment sessions because "I like to be in control of how it's going." It appears that for Sara, lesson planning is important because it calms her fears of encountering the unexpected. By preparing for possible student responses and structuring the discourse around the task prompts, Sara has a sense of control over the lesson. Anticipating student responses enables Sara to prepare for how she will respond to them, again enabling her to control the content of the classroom discourse. Much like her comments about co-teaching, Sara plans for how she will keep the conversation on the prepared task.

Sara's comments reveal a lack of tolerance for discomfort when teaching. She values the autonomy of the enrichment sessions because then she can control every aspect of the lesson – from the task selection to how the lesson unfolds. By exacting control, she hopes to avoid any deviation from her planned lesson, and thus avoid any surprises.

Sara's Selected Tasks

Sara's lack of tolerance for discomfort prompted her to select tasks from reputable mathematics websites, which were recommended by me, the course instructor. She consistently relies on resources from NCTM, Balanced Assessment, and Inside Mathematics. Sara's task choices and lesson plans ensure that during the lesson she will be able to provide correct explanations of mathematical content. Her selected tasks were accompanied by teacher resources such as answer keys with multiple solution paths, possible student misconceptions, and questions to pose to students while working on the task. Equipped with these additional resources, Sara was able to anticipate student responses that differ from her preferred solution method.

Sara stated that her goal was to find "something that I felt, really, that I knew well enough comfortably to answer any questions that they might've had" (Interview 1). Sara used the tasks as presented online, occasionally with minor changes that did not impact the content of the task (e.g., changing the context). When asked to elaborate on how she selected her tasks, Sara said:

> Sometimes, like, if I could take it and I could explain it further or if I could think of a different way to think of it, then I would use it. If it was something that I would look at and I would be like, "I don't know," like, I would get confused on how they got there or I didn't know how to apply it, then I would kind of skip over

> it. Because like I said, I don't want to steer them in the wrong direction. (Interview 1)

Again, Sara's task selection is motivated by preventing any discomfort during the lesson. Sara avoided any tasks that she felt that she could not accurately explain, for fear that she would harm students' mathematical understanding. There were two tasks that Sara felt the most confident in her content knowledge:

> And I liked the Scale Stepping too because...I don't know I just find that like, this is one of the things that I remembered really well from school. So like, like I said, I was more comfortable with teaching it or talking about it because I thought that I could explain it a little better. And this one (Cork Costs) - In this one I was actually taking [Geometry] last semester so I remember doing this one and thinking, "I can apply what I just learned! And I can use it!" (Interview 1)

Sara identifies the Scale Stepping and Cork Costs tasks (both NCTM Problems of the Week) as addressing content that she felt comfortable with. Scale Stepping addresses measures of central tendency, a concept that Sara assesses herself as remembering "really well." Cork Costs asks students to determine the cost of different cork shapes based on their area, a concept that Sara recently revisited in her undergraduate Geometry course. Her selection of these tasks is motivated by her perception of her content knowledge and pedagogical content knowledge.

For Sara, avoiding surprises during instruction includes controlling how the lesson unfolds. The typical structure of Sara's lessons was for students to work on the task independently and then go through each portion of the task as a group. For NCTM Problem of the Week (nctm.org/pows) tasks like "Card Covers" (Lesson 1) or "Counting

Chicken Wings" (Lesson 2) where the task is a single prompt, Sara's plan was to discuss as a group "which solution path we think will be the simplest one to execute" (Lesson Plan 2). For tasks with multiple prompts, Sara's plan was to walk through each prompt together:

> I have noticed that the best way for the students to work through our lessons is working together as a group and allowing them to work through portions of the problems independently. To facilitate participation from all my students, I will call on them specifically to answer a question or have them read portions of the worksheet out loud. This allows each student to participate and lets me know that they are paying attention and understand how we are working through the lesson. (Lesson Plan 5)

Sara's lesson plans have a rigid structure that allow her to maintain control of the discourse, and thus the content: she presents one prompt at a time, asks students to work independently, and then goes over the prompt as a group before moving on to the next prompt. For Sarah, preparing questions and anticipating student responses is an important part of lesson planning:

> [Lesson planning is] also nice because, like I've said, when you have to kind of anticipate what the students are going to say, it gives you time to, like, prepare for that. Because sometimes, if you don't, and the student says something, and you're like, "I have no idea how to address this, I don't know what to do." I think that that's important. (Interview 2)

While Sara's lesson plans may be rigid, she demonstrates an attempt to engage in the Five Discourse Practices, which is evident in her selection of tasks with multiple solution paths, anticipating student responses, and planning for how student discourse will be structured.

Jessica's Use of Course Materials

Jessica's aim of "I want to avoid any surprises" resulted in utilizing tasks from the methods course, from mathematics coursework, and tasks developed by her peers. Like Sara, Jessica's participation with instructional materials is reinforced by her perceptions of teaching and her tolerance for discomfort when teaching. By selecting tasks that she has experienced as a learner, or by consulting with her peers, Jessica is able to anticipate how the task will unfold with her students.

Table 8.

Task Sources for Jessica's Lesson Plans

	Task Source						
	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7
Jessica	Pinterest	Methods Course	Stats Course	Methods Course	Peer	MARS	Peer

Jessica's Personal and Intellectual Resources

Jessica communicates an operational invariant of "high cognitive demand tasks communicate high expectations." In her final paper, Jessica wrote about her understanding of cognitive demand and her attempts to select high cognitive demand tasks with her small group of students:

> As a teacher, you can construct a productive classroom in many different ways. The first step is to set high standards for your students. This semester we talked a lot about making sure that tasks are of high cognitive demand… Learning about these levels of demand also made me realize that it is important to push students

> past their comfort zone and implement high-level tasks. This not only promotes a growth mindset, but also lets students know that you believe they can succeed at higher-level tasks…I ensured that my launch supported the high cognitive demand of my tasks. These tasks fell under the aforementioned categories of "procedures with connections" and "doing mathematics."

In this excerpt, Jessica links the concept of cognitive demand with her perceptions and beliefs about teaching. Her reflection indicates that the features of a "productive classroom" include high expectations for students, as demonstrated by fostering a growth mindset and teaching with high cognitive demand tasks.

Competing with Jessica's belief that she needs to select high cognitive demand tasks is her tolerance for discomfort when enacting tasks. When asked why she chose tasks from coursework, she stated:

> [F]or me, my biggest fear of being a teacher is like, having a worksheet that's pre-made, and not like knowing how to do things myself. And I think it's so important to like, solve everything out and have your own answer key. And that's why I chose stuff that we had done in class. That's why [the MARS tasks] was so hard for me too, is because I was struggling with it. But the rest of the lessons that I chose, I was super comfortable with them. And that's how I feel like you should be when you're a teacher, like you should know exactly how to do these lessons, because your kids are going to throw random questions at you that might be curveballs, and you gotta know how to catch them.

Note that Jessica's choice to select tasks from coursework is not based on the cognitive demand of the tasks, but her sense of comfort with the tasks. Jessica states that she was comfortable with these tasks because she had experienced them as a learner in her courses and knew "exactly how to do them." Like Sara, Jessica wants to be prepared for students' questions and different solution methods. In contrast to Sara's reliance on choosing mathematical content that she felt she knew well, Jessica talks about the comfort and expertise she derived from her experiences with the specific tasks from her courses.

In addition to selecting tasks from coursework, Jessica selected tasks that were created by her peers. As previously mentioned, Vincent had the idea to set-up a shared online drive where students in the methods course could upload their lesson plans. The benefit of sharing lesson plans is that PSTs can discuss how the lesson went when they implemented it with students and offer advice for how to potentially modify the lesson for future use. Jessica implemented a task created by Elizabeth, which was based on a vignette we read in the methods course, and a task created by Vincent, which was a guided discovery of exponent rules. When using Elizabeth's task, Jessica included this note in her lesson plan:

> PLEASE NOTE: most of this lesson plan was copied and pasted directly from Mary Jane's adapted version of Elizabeth's lesson plan. I do not take credit for all parts of this plan; however, I did change around some of the lesson to adapt it for my students. I kept a lot of useful things on the plan that I felt would benefit my own students. (Lesson Plan 7)

In her interview, Jessica stated that sometimes it was difficult to know how to adapt others' lesson plans to fit the needs of her students:

> And one thing that was difficult about that, especially when Mary Jane and I kind

> of shared lesson plans was like, something that I talked to you about is like, how do I make this my own, while also keeping the stuff that I thought was important…But like, it was, it was difficult adapting it to my students. And I would kind of like, hold on to some of the things she had written on her lesson plan and like, want to put that in mine word for word. (Interview 1)

Evident in Jessica's response is her belief that lesson plans should be adapted to the different groups of students. Although Jessica did not modify her peers' tasks in anyway, she did adjust their written lesson plans to better fit her students and her teaching style (e.g., how the discussion is structured).

Jessica's Selected Tasks

In this section I will highlight a few of Jessica's tasks and how her experiences as a learner informed her preparation of the task and the resulting lesson plan. For her second lesson, Jessica selected The Baseball Task, which is a task that we explored in the methods course as an example of a task with the cognitive demand of *procedures with connections*.

The underlying mathematics of the Baseball Task is systems of equations (8.EE.C.8), although it can be solved without symbolic algebra. In the methods course, all PSTs initially approached the task by writing a system of equations, then solved using substitution or elimination. PSTs were prompted to find solution methods that did not require solving systems of equations algebraically or by graphing. PSTs were able to identify two additional solution methods. In the methods course, we discussed when in the learning sequence a teacher would choose to use this task. I shared my experience of using this task to introduce the concept of systems of equations with Math 8 students. The

Baseball Task has the potential cognitive demand of “procedures with connections” when the different solution methods are linked together explicitly.

Jessica was one of three PSTs who implemented The Baseball Task with her students at Bumblebee Middle School (Grace and Elizabeth were the other two PSTs). Jessica’s lesson plan for The Baseball Task reflected the discussion from the methods course. In her lesson plan, she included the four different solution methods that we discussed in class as potential student responses. Jessica also included the potential student misconceptions that were discussed in class (for example, dividing the package cost by the number of items to find the cost of each item). Jessica prepared for this lesson with the expectation that her Algebra I students we already familiar with writing and solving systems of equations. Thus, she was surprised when students did not employ these methods:

> I was able to successfully re-explain to students how to use substitution and elimination to solve the “At the Baseball Shop” problem. I was surprised that my students did not attempt to use this approach on their own since they have learned these methods previously. However, I enjoyed seeing the alternative ways that each student attempted to solve the problem. When the students were finished working the problem out on their own and sharing their own responses, I introduced the substitution and elimination method. After explaining how to solve the problem with this method most of the students had an “Aha!” moment and remembered that they have used substitution and elimination before. I was glad that I was able to refresh their memories and that they understood this alternative approach to solving the problem. (Discussion Post Oct 10)

When students did not organically employ the substitution or elimination method, Jessica showed students how to use these methods to solve the task. In the end, Jessica was able to successfully recreate what was modeled in the methods course by linking the solution methods to one another. Ultimately, her sense of comfort was maintained because of her prior experience with the task.

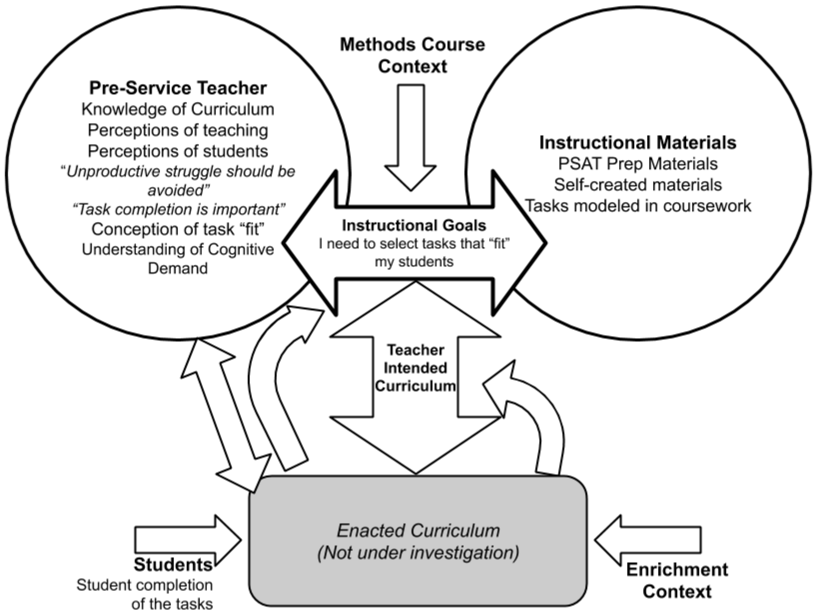
Another task that Jessica selected from coursework was the Surprising Squares task. In the methods course, we watched a series of classroom video clips of a teacher enacting the Surprising Squares task with her students. PSTs first completed the task themselves, then we watched how the teacher launched the task and supported students working in small groups. As a class, we took note of the different questions posed by the teacher and her use of discourse moves. In her lesson plan, Jessica wrote that she chose the Surprising Squares task to use with her students because it addresses “so many of the SfMPs!” (Standards for Mathematical Practice), indicating that she saw this task as an exemplar. Jessica’s lesson plan incorporated both the actions of the teacher from the video and the discussion from the methods course. For example, Jessica planned to use the same launch as the teacher in the video, “What comes to mind when you hear the word pattern?” (Lesson Plan 4). In addition, the potential errors and misconceptions identified by Jessica encompass the errors that students made in the video: “Students might be confused about what an ordered pair is”; “Students may have difficulty with creating a scale for their graph and deciding what their x and y axis will be” (Lesson Plan 4). Jessica’s lesson plans for both The Baseball Task and the Surprising Squares task seem to indicate that selecting and enacting tasks accurately means enacting the tasks as they have been modeled in coursework, whether by me, the course instructor, or by a teacher in a video. Thus, how tasks are modeled for PSTs can influence how they plan to implement the same tasks with students.

The Search for the Best Fitting Task

Nearly every PST described, either in a written reflection or in the interviews, about searching for tasks that are the “right fit” for students. Even with the shared instructional aim of “I need to select tasks that ‘fit’ my students,” PSTs varied in how they defined or assessed task fit. For many PSTs, a task was a “good fit” if it was the right balance of easy and hard, often measured by students engaging in productive, rather than unproductive struggle. The fit, or appropriateness of a task, was also assessed by whether students could successfully complete the task in the allotted time. Hence, the aim of “I need to select task that ‘fit’ my students” is informed by students’ performance on and engagement with prior tasks. PSTs’ conception of task fit is informed by their understanding of methods course content (i.e., productive struggle, cognitive demand), their knowledge of the curriculum, and their perceptions of students (Figure 6).

Figure 6.

Instructional materials and personal resources leveraged as PSTs seek the "best fit" task



Balancing Easy and Hard

According to PSTs, one of the markers of a "good fitting" or well-aligned task is the balance of easy and hard. While interacting with instructional materials, PSTs sought tasks that were both accessible to students and engaged them in productive struggle. Jessica and Elizabeth commented on their desire to select tasks that were "balanced":

> I need to work on finding balance between challenging them just enough that they can complete the task with a small push in the right direction and not giving them work that it is too straightforward. (Jessica, Discussion Board Post, Oct 25)

> [This task] was [on the] high end where they were challenged, but it was still low enough that they could do it. And I think that was something that I was struggling with throughout the semester - trying to find that balance. (Elizabeth, Interview 1)

Embedded in both Jessica and Elizabeth's comments is the idea that tasks should be challenging but accessible ("low enough"). Boaler (2016) refers to tasks of this type as "low floor; high ceiling" (p. 62). While this exact phrase was not employed in the middle grades methods course, it is possible that Jessica and Elizabeth encountered this phrase and concept in their prior early field experience courses or mathematics courses. Another possibility is that PSTs could be connecting to the concept of zone of proximal development, which is discussed in several pre-requisite courses.

Mary Jane refers to the search for finding tasks that were not too easy or too hard as determining "The Goldilocks Zone":

> I feel like one of the biggest things that I cared about when choosing activities was like, like, I wanted to find the Goldilocks, not too easy, not too hard, because, like, one of the things I really didn't want for my students was for them to like, go through unproductive struggle. (Interview 1)

> you always need to find like the Goldilocks zone, you don't want to make your lesson plan too easy to the point where students are disengaged because they're like, 'I already know this, like what's the point,' but also you don't want to make it hard to the point where they're like, 'I'm never gonna - I'm never going to get to the answer. I feel like all my efforts is going to go to waste.' (Interview 2)

For Mary Jane, “The Goldilocks Zone” is marked by the absence of unproductive struggle – but not necessarily the presence of productive struggle. For Mary Jane, Elizabeth, and Jessica, task fit is also determined by whether students are able to complete the task: “get to the answer”; “they could do it”; “they can complete the task.” In the next sections, I dig deeper into PSTs’ metrics of (un)productive struggle and task completion as indicators of alignment between tasks and students.

(Un)Productive Struggle

In their final course papers, six of the ten participants wrote about productive struggle as a necessary and beneficial aspect of learning mathematics. For example, Vincent wrote, “While seemingly counter-intuitive initially, it is this productive struggle that challenges students to ask questions, think critically, and solidify their understanding. As a teacher, it is my job to support this productive struggle.” PSTs identified productive struggle as something to cultivate and unproductive struggle as something to avoid. I interpret PSTs’ statements around productive and unproductive struggle as the operational invariant “unproductive struggle should be avoided.”

Unproductive struggle can be avoided by selecting tasks that leverage students’ prior knowledge or by lowering the cognitive demand of the task. PSTs’ interpreted students’ unproductive struggle with tasks during enrichment as a misalignment between the cognitive demand of the tasks and the students. There was a shared concern among some PSTs (Briley, Carson, Grace) that when the cognitive demand of the task is “too high” students will descend into unproductive struggle. For example, Carson stated, “keeping the demand high but also not too high. Like, if it's too high then they won't get it at all. So, making sure it's productive struggle struggle rather than non-productive struggle" (Interview 2). Thus, while Carson identifies maintaining high cognitive demand as a goal, this should not supersede the goal of students "getting it" through productive struggle.

Related to maintaining the cognitive demand of the task is supporting students in accessing and applying their prior knowledge to the task. Briley concluded that her students' lack of prior knowledge led to unproductive struggle with tasks: "This lesson plan had a high cognitive demand, but it did not go as planned because the students were struggling past a productive level; they didn't have the background knowledge necessary to complete the task" (Final Paper). In fact, in their reflection posts, several PSTs wrote about being surprised that students did not have the prior knowledge that they expected them to have, like Claire, who wrote, "I was in panic mode when they said that they had no experience working with percents since the entire lesson was literally on percents" (Discussion board post, Oct 25). When students lacked the prior knowledge necessary to engage with the planned task, PSTs resorted to direct instruction, which lowered the cognitive demand of the tasks.

When students showed signs of unproductive struggle, Grace found it helpful to "take it a step back":

> [S]ometimes if it is too cognitively demanding and they're getting frustrated and too flustered I think sometimes it'd be like, helpful to take a break and be like, 'Okay, so maybe like, what do you guys remember about this? What are some aspects that you do know?' (Interview 2)

In the quote above, Grace suggests some questions to pose to students to activate their prior knowledge related to the task. These questions mirror the questions modeled in the methods course when learning how to effectively launch a task (Jackson et al., 2012). Grace's suggestion is to revisit those launch questions when students start to devolve into unproductive struggle while working on the task.

While most mentions of productive struggle by PSTs were about how unproductive struggle is an indicator of a misaligned task due to too high cognitive demand, Grace showcased a moment when her students were engaged in productive struggle:

> I could see that the students were struggling with reaching an answer, but they were not stuck, they were constantly coming up with new methods and ideas on how to solve this problem. After a while they got the answer of how much each item cost and they both were so proud of themselves. Then after I had a conversation with one of the boys who said that this had been the hardest lesson yet, but it was his favorite! He said that he really liked being challenged in math and it made him think more and that it was a lot of fun to figure out the prices. (Final Paper)

Grace's 6th grade students successfully navigated the Baseball Task, which was a task from the methods course. Undergirding PSTs' statements about (un)productive struggle is the belief that productive struggle will lead to students successfully solving the task, while unproductive struggle results in students giving up, or otherwise being unable to complete the selected task. Interestingly, although PSTs write about the importance of supporting students in productive struggle, they still seem to value the successful completion of the task (arriving at a solution) over what they can learn about student thinking while working on the task (regardless of completeness).

Task Completion

Another criterion for how PSTs evaluated if a task was a "good fit" for their students was if students were able to successfully complete the task. For PSTs, a task has been successfully completed if students are able to arrive at the correct solution. PSTs' interpreted students' engagement with and interest in the task to contribute to task completion. When students did not successfully complete the mathematical tasks, PSTs attributed students' failure to solve the task to lack of prior knowledge. In summary, PSTs communicate an operational invariant of "it is important that students complete the task."

As discussed in the previous section, there were times when PSTs determined that students were unable to complete the tasks because of not having the requisite prior knowledge. Prior to preparing lessons for the enrichment sessions, PSTs were provided with a list of the CCSS content standards that Bumblebee Middle School courses that would be addressed during the first quarter, according to the district pacing guide. As the course instructor, I encouraged PSTs to select tasks that addressed these first quarter standards, to reinforce what students were learning in their current coursework, or to select standards from the previous grade-level, to refresh and deepen students' understanding of concepts. As the semester progressed, PSTs increasingly chose their lesson topics based on their students' requests. PSTs directly asked students what topics they wanted to focus on or what they were currently learning in their math class:

> At the end of every single week, I would ask my students like, what is something that you're learning in class right now? And what is something that you want to go over next week? And so they would like, tell me and I would take note of it. And

> then I would try to go home and find a lesson plan around that. (Mary Jane, Interview 1)
>
> I started asking them more and more what they were interested in, what were they doing in the math classes during that period, so they'd have that prior knowledge and come in ready. (Vincent, Interview 1)

By asking students about what they were currently learning or struggling with, PSTs were able to select or design tasks that reinforced mathematical concepts. Jessica strategically selected tasks that addressed “stuff that they probably know, so that they have that confidence and that self-efficacy” (Interview 1). By selecting content that was familiar to student, Jessica wanted to ensure that students could complete the task, thus reinforcing their mathematical disposition.

PSTs identified student engagement or motivation as factor for why they were able to complete the task. Student engagement during the after-school enrichment sessions was a priority for many of the PSTs. They were concerned about holding students’ interest for 90 minutes after a 7.5-hour school day. When students showed signs of not being engaged with the selected tasks (e.g., off-topic conversations), PSTs saw this as a defect of the task or their implementation. Engagement with a task is indicated by students’ interest or excitement about the task. Motivation is indicated by students’ willingness to complete the task. PSTs expressed that when working on tasks, students’ interest fuels students’ motivation. Asking students what content they wanted to work on was one way that PSTs tried to increase students’ motivation during the enrichment sessions.

Mary Jane and Grace saw task completion as a catalyst for future engagement. Like Jessica, Mary Jane sees task completion as a way to strengthen students' mathematical dispositions:

> Once the students learn how to do that task or activity, they're now kind of engaged in the learning and want to learn more about like why math is the way it is, rather than like, "oh, let me complete this sheet so that I know how to do it." It's like a different way of engaging and incentivizing students for learning. (Interview 2)

Here, Mary Jane contrasts the engagement that comes from a pattern of success with tasks versus completing worksheets because they must. Similarly, Grace recognized that her activity on adding and subtracting signed numbers was "not challenging" but decided that students still enjoyed it "because they got it pretty fast" (Interview 1). While completion of tasks may increase students' motivation to complete future tasks, Claire argued that "easy" tasks are not always a catalyst for engagement or motivation. Like productive struggle, Claire saw students' motivation to complete the task as linked to the perceived difficulty of the task:

> Because if you like, if you do get something too easy, like they lose motivation… and if you give them something too hard, then they just like, give up because they don't have it. So, if you give them something just challenging enough that they have to think a little, I mean, or a lot, but like if they really think about it, then it really helps them, like want to complete a task. (Interview 2)

PSTs' descriptions of engagement and motivation differ from how "fun" was conceptualized within the instructional aim of having fun with mathematics. What made tasks fun was defined by PSTs' own perceptions of fun – gamification, student choice, contextual features, and personal interests. Here, student engagement is measured by observable behaviors from students and by asking students directly what they did or did not enjoy about the task.

Final Thoughts on Task Selection

Although identifying the cognitive demand of tasks was a focus of the methods course, few PSTs explicitly identified the cognitive demand of tasks as something they considered when looking for and selecting tasks for the after-school enrichment program. However, during the interviews, I noticed that one way that PSTs conceptualized cognitive demand was a measure of student ability (for additional misuses of cognitive demand, see Anthony et al., 2020). The idea of cognitive demand as tied to student ability, rather than as a feature of tasks, did not surface during the course, or in the course assignments. However, it is possible that PSTs held this conception during the course, while selecting tasks and planning lessons. Thus, this misconception of cognitive demand of tasks was an unintended outcome of the course.

In this section, I focus on one PST, Briley, unpacking her misunderstanding of cognitive demand, how it may have been reinforced by her task selection. Then, in contrast, I offer Claire's experience with task selection and how it informed her understanding of the cognitive demand of tasks. Both Briley's and Claire's understandings of cognitive demand can help to inform how cognitive demand is addressed through task selection in the middle grades mathematics course.

Briley

Some PSTs decided that the tasks they selected for enrichment at Bumblebee Middle were just too challenging for their students. This was most salient for Briley, who created all but one of her own tasks for her small group of students. Throughout the semester her tasks often degraded into direct instruction during implementation, due to Briley's assessment that her students lacked the prior knowledge or procedural fluency needed for the task. Briley evaluates students' lack of success in completing the planned tasks as a mismatch between task and students:

> I think they were good lesson plans. But then when I actually tried to use them with the students that I had, like it wasn't as effective, because they weren't like, able to keep up with everything that I had planned… they were really struggling with the stuff that I was coming up with. (Interview 1)

According to Briley, her lesson may be a better fit for a different group of students: "I was thinking about that, how something that is really hard for them is not going to be the highest level cognitive demand for another set of students" (Interview 1). Here, Briley conflates the cognitive demand of the task with the difficulty with the task. Yet, Briley is correct that the cognitive demand of tasks is context dependent – as mathematical process or procedure becomes routinized, tasks that imply that process decline in cognitive demand. However, Briley makes additional remarks about cognitive demand which imply that she sees cognitive demand as a quality of students, not tasks:

> Sometimes I found it hard to maintain high cognitive demand for the students I was teaching. Now, I know that cognitive demand can be respective to the

> intended audience, or students, that you are teaching. What may be lower demand for some students, could be high for others. (Final Paper)

> But I've always struggled with the cognitive demand thing just because in my brain, there's like, it's some sort of disconnect between…it's more individual than I assumed it would be... I am having trouble saying what I'm trying to say. But like, like in my mentor's room I'm gonna have to realize that like cognitive demand for them is going to look different than it is in Grace's mentor's room. (Interview 2)

In the first quote, Briley is reflecting on her experiences with students in the enrichment program at Bumblebee Middle. Again, Briley seems to be conflating cognitive demand with difficulty. Briley's statement also implies that while her tasks may have been of lower cognitive demand, these tasks were still difficult ("high") for her group of students. In the second quote, Briley is reflecting on her current field experience. She compares her mentor's classes (math support classes for students identified as special education or English language learners) to Grace's mentors' classes (Math 8 and Algebra I classes). Briley states that cognitive demand is "more individual" than she previously thought.

One possible explanation for Briley's misunderstandings around cognitive demand is that during enrichment her group of students were not effectively engaged with higher cognitive demand tasks. As shown earlier in this chapter, Briley's tasks tended to be procedurally focused within the guise of fun activities. Although I evaluated four of Briley's tasks as procedures with connections, during implementation the cognitive demand of the task often degraded. Thus, Briley's students had limited opportunities to engage with tasks of higher cognitive demand. Given that Briley created almost all her own tasks and activities, she may have taken it personally when students struggled, and the lesson did not go as planned. Since Briley sees herself as a "creative person" (Interview 1), her sense of self may have been challenged when her lessons fell short of her expectations. It is possible that enacting a different selection of tasks may have resulted in Briley developing a different understanding of cognitive demand. In a later section of this chapter, I will elaborate more on how Briley's experience can inform the methods course specifically and teacher education more broadly.

Claire

In her first interview, Claire stated that over the course of the 7 lessons, the cognitive demand of tasks became an important consideration. Compared to the other PSTs, Claire has the greatest variance in the cognitive demand of her selected tasks (see Table 9 and Appendix J).

Table 9.

The Cognitive Demand of Claire's Selected Tasks

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7
Cognitive Demand of the Task	With Connections	Doing Math	With Connections	Without Connections	Memorization	With Connections	Doing Math

For most of the enrichment program, Claire's selection of tasks appears to be reactionary. As documented in her discussion board posts, Claire attributes students' struggle to a lack of prior knowledge, especially regarding mathematics vocabulary. During enactment, Claire found herself leading students through the tasks. In response to students’ unproductive struggle with prior tasks, Claire overcorrected and for her fifth lesson created a task that is underwhelming – which became a catalyst for shifting her priorities.

Claire’s fifth lesson focused on writing and interpreting ratios. She created this lesson herself and acknowledges in her interview that this lesson was of low cognitive demand (Memorization): “And I remember [the ratios lesson] just wasn't very challenging. Like it just really did not have much of the higher order thinking” (Interview 1). Following this comment, I asked Claire whether she considered cognitive demand when preparing lessons for Bumblebee Middle, she replied:

> Definitely over time. I felt like before I picked mostly tasks that like, they weren't maybe the hardest, like the most cognitively demanding, but once I did this lesson and I saw how not cognitively demanding it was, then it became more priority for me when picking the lessons to make sure that criteria was also being met. (Interview 1)

Although Claire’s response shows that she may conflate difficulty with cognitive demand, her final two lessons were indeed designed around tasks of higher cognitive demand. In her final paper, Claire wrote:

> By the end of my tutoring, I had found a very high-level demand task for my students to work through. The students not only completed the task, but they led the way for most of the discussion. As an educator, I have grown significantly in this area.

Claire assesses her final task as a success. Moreover, she states that this is her favorite lesson: "I think that this was the best one…This is the one that I felt like I really did understand at that point what to do and the task was like really good" (Interview 1). Claire communicates a confidence in her ability to select a high cognitive demand task that is accessible for her students.

Moving Forward

I highlight the perspectives of Claire and Briley to showcase two very different experiences with lesson planning in the methods course, which ultimately contributed to different understandings of cognitive demand. The PSTs in this study demonstrated difficulty with internalizing the descriptions of and applying the four categories of cognitive demand to categorize tasks (Anthony & Viviani, 2020). As documented in reflective writings and the interviews, the PSTs tended to conflate cognitive demand with mathematical difficulty, and misinterpreted surface-level features of tasks as indicators of higher cognitive demand (for example, pseudo real-world contexts). These outcomes indicate that the language of cognitive demand, as presented in the methods course, is not accessible to PSTs. There is value to teaching the categories of cognitive demand explicitly in the methods course, primarily because of the pervasiveness of the language of cognitive demand in mathematics education literature, specifically practitioner-focused publications from NCTM. However, Boaler's (2016) description of challenging but accessible tasks as "low floor, high ceiling" (p. 62) may be more easily applied by PSTs when first learning about tasks of higher cognitive demand. As shown earlier in this chapter, the concept of a "low floor, high ceiling" task was articulated by PSTs when assessing task fit for their students. Thus, being introduced to this language explicitly may support PSTs' understanding of what makes a rich, rigorous mathematical task and eventually bridge their understanding to the formal definitions of cognitive demand.

Chapter 6: Discussion and Implications

I begin this final chapter with a summary of the research study. Then, I highlight some takeaways from this study, beyond the instructional aims and operational invariants identified in Chapter 5. I explore implications of the findings of this study for teacher education, specifically middle grades teacher preparation. Finally, I describe possibilities for future research on PSTs' participatory relationship with instructional materials.

Summary of the Study

This study examined the mathematical tasks selected and created by PSTs when designing lesson plans for middle grades students in an after-school mathematics enrichment program. My goal in undertaking this study was to understand what informs and influences PSTs' selection of tasks within the context of the mediated field experience methods course, where PSTs have autonomy over task selection. Thus, my research questions were:

1. What types of tasks do PSTs select for the after-school enrichment program?

2. What does PSTs' task selection and lesson planning reveal about their instructional aims and their personal and pedagogical resources?

Analysis of PSTs' selected tasks showed that a little more than half (53%) of all the selected or created tasks were of higher cognitive demand. Despite the amount of higher cognitive demand tasks, Appendix J shows that two PSTs, Jake and Grace, did not select or create any tasks of higher cognitive demand. The results also showed that 9 of the 10 PSTs selected at least one task from a not-so-trustworthy website (e.g., TPT, Pinterest) which mirrors the results of Shapiro and colleagues (2019), where 89% of participants regularly utilized at least one not-so-trustworthy website. Finally, approximately one-third of PSTs' enrichment tasks were self-created, which tended to be of lower cognitive demand. Although few PSTs modified existing tasks, the fact that self-created tasks tended be of lower cognitive demand could indicate that when PSTs modify tasks, they may do so in ways that lower the cognitive demand of the task.

Analysis of PSTs' tasks, lesson plans, reflective writings, and interviews provided evidence of their instructional aims, operational invariants, and resources that were activated and leveraged during task selection and lesson planning. Chapter 5 captured the shared instructional aims of PSTs as well as identifying operational invariants and beliefs that influenced PSTs' execution of those aims. I acknowledge that what I have presented is merely one set of outcomes and interpretations of this rich data set (Gueudet & Poisard, 2019). In analyzing the data, I implicitly adopted an asset-based and resources-perspective to teacher learning. A different researcher with a different philosophical lens may have identified different operational invariants and instructional aims. This study is a snapshot of novice teachers who are transitioning from learners and doers of mathematics into mathematics educators. In this final chapter, I discuss how the instructional aims of students having fun, wanting to avoid surprises, and finding tasks that "fit" can inform both future iterations of the methods course and teacher education more broadly.

Concluding Thoughts on Task Selection in the Methods Course

As shown in Chapter 5, the most prevalent instructional aim shared among PSTs was selecting tasks that "fit" their students. This aim is significant for two reasons. First, because PSTs communicated a sense of responsibility to these students and an ownership of their teaching ("my students"). Second, because selecting, modifying, and creating lesson materials that meet the needs and interests of students is a core practice of teaching mathematics.

Ownership of Practice

Working with the same small group of students across 7 lessons helped PSTs to develop relationships and rapport with students that led to PSTs seeing the students as "their students." PSTs recognized the importance of developing relationships with their students. For example, Jessica wrote, "knowing that my students felt comfortable enough with me to open up meant the world and was a huge success in my book" (Discussion board, Nov 29). The opportunity to develop relationships with middle grades students is an asset of the after-school enrichment program. The mediated field experience methods course operates as a third space in the teacher preparation program (Zeichner, 2010). In the after-school program, PSTs have the support of a university teacher educator (me, the course instructor) and a practicing middle grades teacher (the teacher who sponsors the after-school program). Since the enrichment program operates outside the boundaries of a particular teacher's classroom, PSTs have the opportunity to experiment with their pedagogies without fear of pacing guides or contradicting the host teacher (although Sara still expressed concerns about this). Operating in this third space enables PSTs to take ownership of the instructional choices that they make for their students. By contrast, PSTs' experimentation with curricula and pedagogies may be limited by mentor teachers who want to manage the learning of "their students" (Patrick, 2013). Thus, during field placements, the PSTs in this study may not experience the same sense of classroom ownership as they did in the after-school enrichment program.

Learning How to Differentiate

In seeking tasks that "fit" their students, PSTs demonstrated that they value differentiating instruction for students. PSTs' reflections on their teaching show that they understand the importance of learning about their students so that they can design instruction to meet students' needs (Teaching Works, 2021). These beliefs about and dispositions toward tailoring instruction for students are aligned with the goals of the methods course and the broader vision of equitable mathematics teaching, as communicated by NCTM (2014). However, the ways in which PSTs assessed task fit (task completion) or adjusted their instruction for students (devolving into direct instruction) did not always productively align with their expressed beliefs.

What is interesting is that while PSTs talked about finding tasks that fit their students, few PSTs modified existing tasks to meet the needs of their students. Along the spectrum of curriculum use, PSTs were more often off-loading (using tasks as-is) or improvising (creating their own) than adapting (Brown, 2009). Out of 70 lessons, only 12 tasks were modified from their original printed form. Some of these modifications were superficial, such as changing the context of a task from "hockey cards" to "Pokemon cards" (as seen in Nicol & Crespo, 2006). Other modifications included adding scaffolding questions (Elizabeth, Carson) or removing scaffolding questions (Claire). I interpreted tasks or worksheets that were created by curating problems from different sources as improvising. For example, when PSTs selected PSAT practice problems from the PSAT Handbook. Even though PSTs are sourcing from existing materials, the work is improvisational, as they are creating custom instructional materials to meet their own goals.

Although PSTs tended to use selected tasks as-is or create their own tasks during the after-school enrichment program, during the second interview all PSTs talked about the value of modifying or adapting pre-existing tasks and lesson plans. Carson expressed that it would be wasteful for teachers to spend time creating their own tasks:

> I don't think there is any reason nowadays to create your own tasks. Like, there are so many resources and so many tasks that your fellow teachers, [or] the internet has. There are just so many ideas out there and and I think it just doesn't really make sense to put in all this work to create your own thing rather than just taking the core idea, someone else's, modifying it to your students and how they will learn best. (Interview 2)

Other PSTs expressed similar sentiments to Carson’s. For example, Mary Jane stated that the existing tasks and lesson plans that teachers have access to “shouldn't go to waste because you want to spend your time making like, all new activities because that time can be more efficiently used for formative assessment to see where your students are at” (Interview 2). While PSTs expressed that modifying existing tasks might be the best use of their time, several PSTs expressed that there are still instances where you may want to search for new tasks or create your own. For example, Jake stated, “finding tasks that teach the same curriculum, but might fit into different interests and different student personalities and different like, classroom demographics. I think that's going to be the most important part of my quote-unquote lesson planning as I get further into teaching” (Interview 2).

Although modifying tasks was discussed and practiced in the methods course, few PSTs modified the tasks that they selected for their lessons. PSTs’ evolving dispositions toward task and lesson plan modifications and adaptations during the second interview may be attributed to their initial field placement and concurrent coursework. At the time of the second interview, PSTs were observing in their field placements at least one day a week and were enrolled in a course that addressed differentiated instruction for academically, culturally, and linguistically diverse students. Given that PSTs had autonomy over task selection during the methods course, they were able to seek out or create tasks that aligned with their instructional aims. However, in their field placements, PSTs and their mentors are working from district-mandated curriculums – although the extent to which teachers must adhere to the curriculums varies by district and school. Regardless, PSTs observed how their mentor teachers adapt the provided curriculum materials or modify their personal lesson plans from previous years. Concurrently, as part of their coursework, PSTs were asked to adapt their existing lesson plans to (1) be responsive to the needs of students with exceptionalities and (2) address the language development of emergent bilinguals in mathematics. Hence, PSTs' recent experiences in adapting lesson plans for different learners as well as observing their mentors' lesson planning practices likely attributed to PSTs' dispositions toward modifying and adapting tasks, as expressed in the second interview.

Implications for Teacher Education

The PSTs in this study expressed shared instructional aims that should be leveraged by teacher educators when supporting PSTs' development of lesson planning practices. In this section, I explore ways that teacher educators can leverage PSTs' productive beliefs and operational invariants. Additionally, I make a case for why middle grades teacher preparation programs should intentionally focus on developing PSTs' self-efficacy.

Promoting Productive Operational Invariants

This study demonstrates how a documentational approach to didactics (Gueudet & Trouche, 2009) can be used to unpack PSTs' participatory relationship with instructional materials and identify resources utilized when selecting tasks and creating lesson plans. By monitoring how PSTs participate with instructional materials, teacher educators can infer PSTs developing operational invariants. Awareness of PSTs beliefs and developing operational invariants can help teacher educators to both purposefully support productive beliefs and seek to disrupt unproductive beliefs and dispositions (de Araujo et al., 2021). In this study, I have identified productive operational invariants and instructional aims that could be further developed through teacher preparation coursework.

Gueudet and Poisard (2019) state that operational invariants are the most important part of teachers' schemes of utilization because they both inform the documentation process (i.e., the selection and creation of materials) and are formed by the process. How teachers (and PSTs) use instructional materials depends on their operational invariants, thus, teacher educators should intentionally incorporate mathematics tasks that align with those operational invariants, making teachers more likely to adopt those tasks (Gueudet & Poisard, 2019). Aligning instructional materials with PSTs' operational invariants can be a way to promote and strengthen the connections between PSTs' productive beliefs and ambitious teaching practices. One way to do this would be by selecting materials that appeal to PSTs' beliefs about fun or the importance of student choice but are also of higher cognitive demand.

Thus, teacher educators modeling the selection and implementation of high cognitive demand tasks is an essential pedagogy for equipping PSTs to seek, select, and prepare lessons using high cognitive demand tasks. In this study, six of the ten participants selected at least one task that was modeled by their instructor in either the mathematics methods course, or one of the mathematics content courses (see Appendix J). These findings highlight the importance of engaging PSTs in high cognitive demand tasks in the mathematics content courses – not only the mathematics methods course. At the time of this study, each of those content courses was taught by a mathematics teacher educator. Given that PSTs designed their lessons after the instruction modeled by their instructors, how these content courses are taught matters. Mary Jane used a task from her Statistics course because she "knew what was supposed to happen" (Interview 1) based on how her instructor implemented the task. Furthermore, Carson saw the tasks employed in his Algebra course as exemplars:

> And those lessons are really really good. And I also like, fully understood them because like, I had to do them myself in the class, so... I did those a few times [with my students] and it was nice that I kinda had that reassurance that, like, "Alright, this was taught to me. So I know it's probably good to teach to them too." (Interview 1)

Carson's comment also communicates a sense of trust about his coursework – that what he is experiencing in the middle grades mathematics and science program is the type of teaching he should emulate.

In addition to purposefully implementing mathematical tasks that appeal to PSTs' developing operational invariants, teacher educators need to ensure that PSTs practice what has been modeled. Gainsburg (2012) found that novice teachers were less likely to use teaching practices that they had not first attempted in a low-risk setting. This finding implies that PSTs who tended to choose lower cognitive demand tasks (such as Jake and Grace) or who encountered challenges when implementing ambitious teaching practices (e.g. facilitating a mathematical discussion) during their enrichment lessons may be more hesitant to implement high cognitive demand tasks with their future students. One way to address this issue may be to require that PSTs implement an instructor-selected task with their students or choose a task from the methods course. Employing these restrictions would ensure that PSTs prepare and enact at least one lesson around a higher cognitive demand task.

More Deliberate Attention to Launching a Task

While the cognitive demand of tasks heavily influences students' opportunities for learning (Stein et al., 2000; Tekkumru-Kisa et al., 2020), research has shown that teachers often struggle with maintaining the cognitive demand of *procedures with connections* and *doing math* tasks during enactment (Cobb et al., 2018; Stein et al., 2000). Requiring PSTs to select a task from coursework may ensure that the task is high cognitive demand, but it does not guarantee that the task will be implemented as such. In

Chapter 5, the section on the instructional aim of "I need to select task that 'fit' my students" captures PSTs' desires to avoid unproductive struggle and to support students in completing the task. In response, PSTs often lowered the cognitive demand of the task. Furthermore, an explanation for the presence of unproductive struggle could be a failed launching of the task.

How a teacher introduces the task to students is a crucial element in sustaining high cognitive demand tasks (Cobb et al., 2018). In a successful launch, the teacher supports students' development of a common language around the task features and cues students to key mathematical concepts without suggesting a solution method (Cobb et al., 2018; Jackson et al., 2012). When Grace's students showed signs of unproductive struggle, she found it helpful to "take a break and be like, 'Okay, so maybe, what do you guys remember about this? What are some aspects that you do know?'" (Interview 2). Grace's questions to students mirror the questions modeled in the methods course when learning how to effectively launch a task (Jackson et al., 2012). Her suggestion is to revisit those launch questions when students start to devolve into unproductive struggle while working on the task. Grace's solution to students' struggle implies that perhaps the initial launch was not successful at activating prior knowledge and creating a common language.

The longitudinal data collected by the Middle School Mathematics and the Institutional Setting of Teaching project (MIST; Cobb et al., 2018) identifies the successful launch of high cognitive demand tasks as a key aspect of ambitious and equitable mathematics teaching. The MIST team found that although the teachers in their study often selected higher cognitive demand tasks, only 6.7% of tasks were successfully implemented at a high level. Teachers consistently lower the cognitive demand of tasks in response to perceptions of student struggle, which disproportionality impacted students of color. Hence, teacher educators must support teachers and PSTs in identifying and practicing ways to support struggling students, without lowering cognitive demand. Such supports include teacher educators' modeling successful task launches and discourse practices, which reinforces my earlier statements about teacher educator modeling in both mathematics content and methods courses.

Attending to Self-Efficacy

In Chapter 5, I profiled Sara, a PST who struggled with self-efficacy around mathematics content knowledge throughout the methods course. Sara's statements express a lack of confidence with mathematics that is often attributed to pre-service and in-service elementary teachers (e.g., Charalambous et al., 2008). However, given that the middle grades preparation program at this university is a dual certification program in both mathematics and science, there are likely other PSTs who feel more confident in their science content knowledge, like Sara.

One way teacher educators can support PSTs in developing their confidence around mathematics content knowledge for teaching is through the use of educative instructional materials (Drake et al., 2014). As shown in Chapter 5, Sara relied heavily on educative instructional materials. All of the resources utilized by Sara were originally recommend by me, the course instructor, and shared with the whole cohort via the course LMS (Appendix A). However, a more intentional approach to incorporating educative instructional materials into the methods course could support PSTs' confidence in both their content knowledge and ability to select rigorous math tasks. As noted by Drake and colleagues (2014), "educative materials provide substantial information for teachers, but leveraging that educative information requires teachers to read and interpret the materials in specific ways" (p. 159). Thus, teacher educators must support PSTs in developing metacognitive strategies for interacting with instructional materials (Sherin & Drake, 2009).

Generalizability

The purpose of this study is not to make broad claims about the personal and pedagogical resources of PSTs. In a different semester, with a different cohort of PSTs, I may have arrived at a different set of conclusions. Furthermore, the organization of the after-school enrichment program at Bumblebee Middle is highly variable. Students, and their parents, self-select into the program and there is a different number of participants across grade-levels each year. Both the size of the PST cohort and the amount of middle grade students in the enrichment program has an impact on how the program is organized. During this study, Fall 2018, there were 18 PSTs, each working with groups of two to four middle grades students. However, in Fall 2019, there were 12 PSTs, each working with groups of three to six middle grades students. Thus, one limitation of this study may be the challenges faced by PSTs when enacting tasks with too few students, which may have resulted in limited opportunities for PSTs to elicit a variety of solution methods or student to student discourse. In addition to variations in the number of middle grades students and PSTs, the learning needs of students also varies by year. During this study, Fall 2018, there were few students identified as ELL, despite the high population of ELL students at Bumblebee Middle. Thus, this study does not capture how PSTs attended to the language needs of ELLs during mathematics lessons.

Despite these limitations, this study does capture patterns of thought that were shared amongst this cohort of PSTs. Some of these patterns, such as misconceptions around cognitive demand, appear to be pervasive among PSTs. Since collecting the data for this study, I have taught the middle grades mathematics course two more times. PSTs across cohorts tend to conflate mathematical complexity or problem difficulty with cognitive demand. In addition, PSTs tend to classify tasks that are verbose or embedded in a pseudo real world context (e.g., calculating discounted sales prices) as being of higher cognitive demand. While tasks of each type of cognitive demand have their purposes in mathematics instruction, the concern is that PSTs will perceive that they are engaging their students in tasks of high cognitive demand when they are not. Thus, I stand by my earlier recommendation of broadening the conversation around categories of cognitive demand to begin with the concept of “low floor, high ceiling.”

Future Research

This study details PSTs’ instructional aims and the personal and pedagogical resources that they leveraged to meet those aims when preparing lessons in their undergraduate middle grades mathematics methods course. Future research could monitor how these instructional aims and developing operational invariants are sustained or altered over time and across contexts. For instance, which of these operational invariants are present when lesson planning for science classes? or, during their full-time teaching internship under the direction of their mentor? While a documental approach to didactics

(Gueudet & Trouche, 2009) can be used to infer operational invariants from lesson plans and other instructional artifacts, I posit that a framing perspective (e.g., Goffman, 1972; Hammer et al., 2005) can be used to examine sets of beliefs, dispositions, and operational invariants that are present across contexts. Frames are a helpful way of thinking about how novice teachers' knowledge builds across contexts, rather than perceiving changes or inconsistencies in PST behavior as a "washing out" of the teacher preparation program (Richards et al., 2020). Both a documental approach to didactics and framing offer a resources perspective to teacher learning.

Throughout the writing of this dissertation study, I wrestled with how to make sense of the participatory relationship between PSTs and the instructional materials. Although Remillard's (2005) framework gave me the language to use, I struggled with how to communicate the interactions between each PST and materials, and how to then make links across those interactions to look for any common behaviors among the cohort. When I consulted with Dr. Chazan, he directed me towards Gueudet and Trouche (2009), who write about teachers' process of documentational genesis. The way that the authors describe how teachers' schemes of utilization are composed of physical artifacts and personal resources that shape how teachers develop instructional materials sounded like framing to me. I ultimately relied on documental approach to didactics because it was the appropriate tool for my data and the story I wanted to tell – shared instructional aims and patterns of thinking across the cohort.

However, I still think that framing would be an appropriate way to monitor the participatory relationship of a single participant over time and across contexts. While a documental approach to didactics and framing share similarities in how they conceptualize a teachers' resources, they differ in their purposes. A documental approach to didactics seeks to identify teachers' operational invariants by looking for consistent patterns of behavior across contexts, while framing seeks to explain variance across contexts. As PSTs transition from the method course to their internship to teachers of record, framing could help to explain the likely fluctuations in PSTs' beliefs, perceptions, and behaviors in regards to task selection and lesson planning across contexts (Richards et al., 2020).

Given the unique context of the middle grades mathematics methods course, I anticipate that different instructional aims and resources would be identified when examining the selected tasks and lesson plans of participants as they transition from PSTs to novice teachers. I posit that at this stage, the beliefs and perceptions that inform PSTs' task selection and lesson planning are more fluid than invariant. One example of PSTs' evolving understanding of task selection is captured earlier in this Chapter, when describing how PSTs expressed the utility of modifying existing (or provided) tasks, despite heavily relying on tasks as-is or creating their own for the enrichment sessions. Within the context of their initial internships, PSTs show evidence of shifting from an aim of "*finding tasks that 'fit'*" to "*modifying tasks to meet students' needs*." In summary, longitudinal research of pre-service teachers as they transition to teachers of record could capture which of the personal and pedagogical resources that inform task selection and lesson planning are truly invariant.

Appendix A: Lesson Planning Resources

Common Core State Standards
http://www.corestandards.org/
http://www.corestandards.org/read-the-standards/
https://turnonccmath.net

Maryland State Standards
http://mdk12.msde.maryland.gov

Financial Literacy Standards
https://www.financialeducatorscouncil.org/financial-literacy-framework-standards/6th-8th/
Lesson Plans - http://www.moneyasyoulearn.org/?grades=6-8&subject=math&bid

PARCC Practice Tests and Public Release Items
https://parcc.pearson.com/practice-tests/

https://prc.parcconline.org/assessments/parcc-released-items?title=&field_subject_tid=Math&field_grade_level_unlimited_tid=All&page=2

Resources for full lesson plans (free)
http://www.nctm.org/resources/default.aspx?id=230
http://map.mathshell.org/materials/index.php
http://illuminations.nctm.org/Lessons.aspx
https://www.youcubed.org
https://www.illustrativemathematics.org/

Social justice focused - http://www.radicalmath.org/
Social justice focused - http://www.tolerance.org

Good Problems/Tasks (but not necessarily lesson plans)

http://www.insidemathematics.org/performance-assessment-tasks
http://figurethis.org/challenges/math_index.htm
http://balancedassessment.concord.org/
http://nrich.maths.org/curriculum (UK resource)
Dan Meyer's 3-Act Math Tasks
https://exploremtbos.wordpress.com/
http://graphingstories.com/

Digital Resources

National Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/vlibrary.html
https://www.desmos.com/
https://www.geogebra.org/
https://www.khanacademy.org/

Big Ideas Math textbook (Used by Bumblebee Middle)
https://bim.easyaccessmaterials.com/index.php?location_user=cc

https://bim.easyaccessmaterials.com/index.php?location_user=cc2

Appendix B: Course Syllabus

UNIVERSITY OF MARYLAND, COLLEGE PARK
Department of Teaching and Learning, Policy and Leadership
Teaching and Learning Middle School Mathematics

Fall 2018

Meeting Days and Time: Tues and Thurs 3:10 – 5:00 pm
Instructor: Monica Anthony
Classroom: 2121 Benjamin
Email: mrd@umd.edu
Office Location: 0313 Benjamin
Office Hours: By appointment

TLPL413- Teaching and Learning Middle School Mathematics (3 credits):

This course focuses on methods of promoting middle grades student learning of mathematics, understanding the conceptual difficulties students have in moving from whole numbers to rational numbers, additive thinking to multiplicative thinking, and engaging in applications of multiplicative reasoning, including connections to geometry and measurement. The course includes developing the core ideas of learning, teaching, and planning for teaching middle school mathematics.

I. Course Purpose:

The goal of teacher preparation programs at UMCP is to prepare reflective teachers for classrooms of diverse learners, through research-based inquiry. The goals for school mathematics that are currently accepted by the mathematics education profession are contained in the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* publication. These are the State Curriculum (SC) of Maryland, as well as the Common Core State Standards for Mathematics, both the content and practice standards. Furthermore, we will engage in discussion about the PARCC assessment. Additionally, we will also make use of the Standards for High Leverage Practices.
The standards will be used as resources for thinking about the goals of the mathematics curriculum. Readings and discussion of material in the text and materials given out in the course provide research findings on what is known about learners' understanding of mathematics and what is known about effective practices for teaching mathematics to middle grade students.

II. Learning Outcomes/Goals:

This course focuses on both the mathematical resources needed for teaching middle school mathematics and the instructional practices of mathematics teaching that will enable you to teach mathematics effectively to *all* young adolescents. This course aims to help you acquire the specialized knowledge and skills needed for teaching middle school mathematics. The course is designed to enable you to support adolescents in their capacity to engage in specific mathematical practices (Common Core Standards for Mathematical Practice):

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Teaching mathematics requires a great deal of specialized mathematical knowledge – knowledge that is different from what it takes to do well in a math course as a student yourself or to be good at other jobs that require mathematics. *Your own understanding, fluency, and comfort with mathematics will be important to your effectiveness as a teacher.* In preparing to teach, you will have to determine the mathematical goals of activities, anticipate the varied ways students might respond, and prepare mathematically for what might happen as a lesson unfolds. You will need to prepare good questions to ask. You will have to generate easier as well as harder versions of problems, either as back-up plans or as ways to focus or extend students' work. As a teacher, you will need a keen sense of the complexity of particular mathematical ideas, and ways they can be scaffolded for students' learning. When your students have trouble, or get answers wrong, you will need to do more than know that they are confused, or that they have incorrect solutions: you will need to be able to figure out what they are doing mathematically and whether it makes sense.

Teaching young adolescents requires knowledge of teaching/learning strategies that take into consideration, and capitalize upon, the developmental characteristics of all young adolescents. You will need to make content meaningful to students and provide learning opportunities that support the intellectual, social, emotional, and physical development of students based on an understanding of childhood development and learning. As a middle grades or high school teacher, you will need to understand that the development of all young adolescents occurs in the context of classrooms, families, peer groups, communities, and society. You will need to understand the importance of establishing close, mutually respectful relationships with all young adolescents, and learn how to create and maintain supportive learning environments where developmental differences are respected and supported, and individual potential is encouraged.

Preparing for teaching mathematics: content, planning, and pedagogy

Mathematical content: Development of deep and flexible understanding of important middle school content such as: ratios and proportional relationships, the number system, expressions and equations, geometry, and statistics and probability. We will strive to "unpack" your understanding of key middle school mathematics topics through the exploration of mathematical tasks during class.

Good mathematics teaching does not happen by chance. Effectively connecting students with mathematics requires planning — deliberate design and preparation. Furthermore, planning matters more than ever in an environment where there are so many demands on teachers. Good teaching depends on being able to manage these multiple demands. To provide high-quality mathematics instruction, teachers need to know in detail both the mathematics and the students they are teaching. Instruction must focus on essential content and skills, be coherent and engaging, and support diverse learners. When planning lessons, teachers consider whether the tasks and examples appropriately represent and engage students in the central mathematical ideas of the lesson. Teachers also think about what students will need to know in order to do the work, anticipate likely misconceptions or confusions, and figure out ways to support students' learning. They consider whether the specific features of the lesson, such as the context or language, are accessible to and support the mathematics learning of all students. Even when teachers have well-designed instructional materials, they still need to plan their lessons to adapt them to district priorities and resources and to specific learner needs. *By the end of this course, you will have developed more sophisticated planning skills that allow you to take into account both your students and the mathematics.*

In order to prepare effectively for instruction you will need to develop good skills in the following pedagogical practices:

Choosing a good mathematical task:
Not all mathematics problems and tasks offer students the same opportunities to develop important mathematical knowledge. In this course you will analyze a variety of math tasks to identify specific features that make them a good task and understand in what ways these features represent an important potential for student learning.

Anticipating students' responses to a mathematical task:
Students, if not given a specific method, solve problems in many different ways, using a variety of, sometimes correct and sometimes incorrect, strategies. In preparing for your teaching it is crucial that you anticipate different strategies and solutions, both correct and incorrect, that your students may use when solving the task you chose for them. The better you can anticipate your students' responses, the better you will be prepared to assess and react to them and help your students learn. In this course you will work with your classmates on anticipating students' responses as part of getting ready to facilitate a small group work in the tutoring sessions.

Planning questions to help students learn mathematics conceptually:
Once you have chosen the task and developed some ideas of ways in which your students may approach solving it, it is important that you also plan how to react to them, including how to help students who get stuck in the process of solving it. Although it is impossible to know what your students will actually do, and therefore know exactly what to do in each situation, planning for hypothetical, but likely responses, will equip you to better react to situations you could not anticipate. The better you anticipate student responses and how to react to them, the better equipped you will be for the uncertainties of teaching. In this course you will learn what questions have the potential to help your students move forward in their mathematical thinking and therefore you will be able to incorporate them in your planning.

Integrating supports for a diverse group of students
Students not only solve problems in different ways, they also take in and process information in a variety of ways that have to do with their experiences and identity. As a teacher you need to be equipped with multiple and flexible ways to present information to

your students and be able to build on their strengths. Also, it is of great importance that you not only allow, but also encourage and support your students in demonstrating their knowledge in any way they can. As part of preparing for teaching you will think about different ways to present a task and any information you need to convey to your students, and how you can modify an activity in ways that do not take away the mathematical learning for your students, but make the task more accessible to them.

Facilitating learning: Understanding and reacting, eliciting and assessing, understanding and responding

Once you have planned your lesson you will have a clear idea of the content you have to teach and how you would like to teach it. However, in practice, things do not happen necessarily the way we planned them. *A good lesson plan should help you be flexible and responsive to your students, and adapt your lesson to their needs.* That is why it is so important in your lesson plan to try to anticipate different scenarios and different ways to approach the task with your students. In order to successfully facilitate your tutoring session, first you need to assess your students' knowledge about the subject, so you can help your students connect the new concepts to their previous knowledge. As you work with your students you will realize that they probably had mathematical experiences different from your own and think about mathematics in different and original ways. *It is your responsibility as a teacher to be open to their different approaches, embrace them and build on them to increase your students' mathematical knowledge and understanding.* One of the most important ways you do this is through asking good questions. Questioning is one of the most critical skills you need to develop as a teacher. It will help you assess your students' knowledge and dispositions, as well as help them figure out how to solve mathematical problems and advance their knowledge and understanding of mathematical concepts.

Understanding and reacting to students' mathematical experiences that are different from your own:

As a teacher, you will need to be aware that students will arrive in your classroom with a wide range of prior mathematical experiences and display a wide range of mathematical dispositions. You will have to understand and embrace your students' experiences and build on them in your lesson planning in order to make the work relevant to your students. By the end of this course, you will have developed strategies that help you observe and listen past your own enculturation and consider the mathematical experiences and resulting dispositions that are different from your own.

Eliciting and assessing students' mathematical knowledge, skills, and dispositions:

Finding out what students know, how they work on mathematical tasks, and their dispositions toward the subject is essential for responsible instruction. Teachers use a variety of assessment practices to improve their teaching, to document their students' achievement, and to inform students, parents, and other educational stakeholders. Assessment encompasses much more than testing and grading. It includes interacting with students as they are learning, pausing to document what students are saying, and noticing patterns in students' work. By the end of this course, you will be able to use multiple techniques to assess students, and make use of the information you gain.

Understanding and responding to mathematical thinking that is different from your own: As a teacher, you will need to understand, evaluate, and react to your students' explanations and representations. In preparing to do this well, it is important that you get accustomed to thinking in multiple ways and to identifying correspondences among alternative explanations and representations. Once you understand your students' thinking process, your goal as a teacher is to help your students learn important mathematics by advancing their knowledge from where it is to more sophisticated mathematical ideas. In order to do that, questioning is likely the most important skill you need to develop as a teacher. Asking good questions requires to deeply understand your students thinking and reasoning behind the gaps in their understanding. Simply asking questions is not the same as asking good questions. Often teachers can easily identify what is not correct in students' work, but the questions they ask do not help students move forward in their thinking. This is because the questions make sense to the teacher but do not necessarily take into account the way students are thinking. In this course you will develop strategies that help you listen past your own enculturation and value the logic of responses that are different from your own, and practice asking questions that help your students unpack their mathematical reasoning and figure out where their and their peers' mistakes and misconceptions are, and therefore learn in a more conceptual way.

Course Activities to Obtain Goals:

- *Participating in a common practice:* Our class activities, discussions, and interactions offer us opportunities to study the practice of teaching from the inside. In this course, you will participate in activities that allow you and your classmates to directly engage in learning. You will also reflect on your own learnings, and what you learn from others. In this way, we will develop insights and knowledge about the work of planning for mathematics teaching and facilitating mathematics teaching.
- *Exploration of mathematics work*: Our class activities, discussions, and interactions offer us opportunities to study practice from the inside. What we do with you is teaching, and you and your classmates are directly engaged in learning. There will be things to learn from reflections on our interactions together. We will work on all the strands of the course, developing insights and knowledge from the work and activities we do together.
- *Learning from classroom practice*: We will collectively study classroom practice to learn the work of teaching. We will use records of practice (e.g., videotapes of lessons, students' work, and teacher's plans and reflections) as a shared text for studying mathematics, the work of teaching, and students. We will examine different frameworks for observations and lesson planning.
- *Enacting practice:* In the tutoring sessions you will have repeated opportunities to engage in a range of instructional activities with middle school students. These activities will be opportunities for your students to learn mathematics and for you to learn mathematics teaching. You will develop your skills over time as you reflect on your experiences and use them to systematically improve your teaching.

III. Preparing for students with special needs

Knowledge of pedagogical strategies and mathematical content and knowing and being skillful with particular teaching practices is only part of what it takes to teach well. Professional teaching also requires the professional judgment needed to make decisions about what to do in specific contexts, with particular students and content. Professional practice is guided by *principles* -- overarching professional commitments, drawn from the values and wisdom of the teaching profession, academic disciplines, and society. Meeting the needs of all students, including exceptional learners, is an essential component of teaching. The professional standards and websites for ELL, SPED and TAG students are: WIDA (World-Class Instructional Design and Assessment) for ELL students, the Maryland Public Schools division of special education and early intervention, and the Maryland Public Schools division of Gifted and Talented. The following three principles will be integrated throughout the course:

> **Providing for Students with Special Needs**: Excellence in mathematics education requires equity – high expectations and strong support for all students (NCTM, 2000). Research has shown that all children can learn mathematics when they have access to high-quality instructional programs that support their learning. Responsible teaching requires accommodating differences to help everyone learn mathematics.
>
> **Providing Students Opportunities to Learn:** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000). Responsible teaching is informed and guided by theories of learning.
>
> **Normative Expectations for Social and Mathematical Behavior:** The teaching and learning that occurs in the classroom is influenced by normative expectations for social and mathematical behavior. By middle school, students enter the classroom with experiential knowledge and deeply held beliefs about the nature of mathematics, about their ability to learn mathematics, and about the roles of teachers and students in a mathematics classroom. Responsible teaching recognizes that new roles and responsibilities must be negotiated, made explicit, and practiced by both the students and the teacher.

IV. Suggested Course Texts/Readings:

There are no required textbooks for this course; however, I ask that you purchase a National Council of Teachers of Mathematics (NCTM) professional membership. We will be using many materials (e.g., articles and lesson plans) published by NCTM. All of these materials can be accessed via the internet with a student e-membership. The student e-membership costs $49 and is good for the entire school year. Please register at the following website http://www.nctm.org/Membership/Membership-Options-for-Individuals/. You only are asked to register for the student e-membership. This will give you electronic access to one journal of your choice (e.g., *Mathematics Teaching in the Middle School* or *Mathematics Teacher*).

For those of you looking for a comprehensive content and pedagogy resource, I recommend purchasing the following:

Van de Walle, J. A., Bay-Williams, J. M, Lovin, L. H., & Karp, K. S, & (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8*. (2nd ed.). Boston, MA: Pearson. (ISBN-10: 0132824868).

This can be purchased for under $50 on Amazon, but again, this is NOT required.

Also, we will look at excerpts from the following text, which can be retrieved from the following website: http://www.nap.edu/catalog.php?record_id=9822

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: The National Academies Press.

V. Course Assignments and Grading:

Weekly Blog Posts

- Each student will maintain a "personal" blog in ELMS, for a total of 15 posts. Prompts are posted in ELMS under "Assignments." Everyone is expected to comment on at least one peer's blog per week. Blog posts and comments are due by midnight (12:00 am) on Sundays.
- Blog posts and comments will be scored according to the rubric posted on ELMS.

Note. Midway during the semester we switched from blog posts to discussion board posts because of issues with the blog software add-on in the LMS.

Mock Interview

- You will be given the interview prompts in class. You will be asked to spend fifteen minutes preparing a response, and then you will be expected to submit a video recording of your response.

Analysis of Questioning Practices

- You will be expected to analyze an audio recording of a tutoring session. This assignment will give you the opportunity to reflect on your questioning strategies and what you learned from the student.

Field placement assignments

- Over the course of the semester we will visit Bumblebee Middle School 8-10 times where you will have the opportunity to work with a small group of 6th-8th grade students. You will have assignments for each of these visits. Some assignments will involve a description of the plans you will be teaching. Other assignments will center on an analysis and assessment of student learning that occurred during your work with these students.

Final paper

- The course will conclude with a final paper designed to focus on your knowledge and skills for teaching mathematics, as well as your pedagogical skills. The final paper is designed to align with the course goals of developing your proficiency as a beginning teacher of mathematics in this course, and to help prepare you for requirements that will be necessary to take the next step in your teaching career.

Attendance and class participation:

Your participation in our class activities and discussions is important not only for your own learning but also the learning of others. Sharing your ideas and questions with the group, as well as responding to those of your classmates, is critical to our work together. As a teacher, you need to do more than understand your own thinking — you must be able to track others' thinking, figure out what others are saying, and determine whether and how they make sense. In our class, the "others" will be your classmates. But in the field and in the future, they will be your students, and sometimes your fellow teachers. Thus, listening to and interacting with others in our class is intended to help you develop dispositions and skills that matter for teaching. Talking in class is also crucial. As a teacher, you will have to speak mathematically all the time. This course provides you the opportunity to learn to speak more clearly, with an attentive focus on your listener. Assignments requiring you to share or present about daily readings are included in the attendance and class participation.

In summary, I expect you to engage in the assigned readings prior to class, attend every class, arrive on time for a prompt start, and participate in and contribute to class. If circumstances prevent you from attending class, I ask that you call or send an email in advance and that you make plans for how you will make up the work you will miss.

Evaluation:

Required activities and assignments are worth points as follows:

Assignments	**Points**	**Due Dates**
Attendance and class participation	54	Daily
Weekly Blog Posts	64	Sundays at Midnight
Mock Interview	18	Monday September 10th (by midnight)
Field Placement Assignments	110	Weekly Starting in Oct
Audio Analysis	20	Nov 13
Final Paper	40	TBD
TOTAL	**312**	

Assignments should be uploaded to ELMS by 11:59 pm (unless noted otherwise). Final course grades will be assigned based on the percentage of possible points earned. The scale used in grading is listed below.

A: 100%-94%	A-: 93%-90%	B+: 89%-87%	B: 86%-84%	B-: 83%-80
C+: 79%-77%	C: 76%-74%	C-: 73%-70%	D: 69%-60%	F: 59%-0%

VI. University Policies

Individual Needs Accommodation: If you have a documented disability that requires course accommodations, please see the instructor as soon as possible. The University is legally obligated to provide appropriate accommodations for students with documented disabilities. In order to ascertain what accommodations may need to be provided, students with disabilities should inform the instructors of their needs at the beginning of the semester. The instructor will then consult with Disability Support Services (314-7682). DSS will make arrangements with the student to determine and implement appropriate academic accommodations.

Religious Observance: The University System of Maryland policy "Assignments and Attendance on Dates of Religious Observance" provides that students *should not be penalized because of observances of their religious beliefs; students shall be given an opportunity, whenever feasible, to make up within a reasonable time any academic assignment that is missed due to individual participation in religious observances.*

We are a diverse community and enroll students of many religions; pursuant to policy, we will do what we can when there are students' requests for excused absences and make- up test requests due to reasons of religious observances. *It is the student's responsibility to inform* the instructor of any intended absences for religious observances *in advance. Notice should be provided as soon as possible but no later than the end of the schedule adjustment period.*

Honor Code: The University is one of a small number of universities with a student- administered Code of Academic Integrity and an Honor Pledge. The Code prohibits students from cheating on exams, plagiarizing papers, submitting the same paper for credit in two courses without authorization, buying papers, submitting fraudulent documents, and forging signatures. Students should write the following signed statement on the top of each examination or assignment: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination (or assignment).* Compliance with the code is administered by the Student Honor Council, which strives to promote a “community of trust” on the College Park campus.

Course Evaluation: As a member of our academic community, you as a student have a number of important responsibilities. One of these responsibilities is to submit your course evaluations each term though CourseEvalUM in order to help faculty and administrators improve teaching and learning at Maryland. Please watch for the dates the system will be open for evaluation of this semester.

VII. Note about Expectations:

Clear, open, and consistent communication is an essential part of any graduate level course. This includes communication between the instructor and students, among students, and in this instance, between you and your various support systems (mentors, supervisors, etc.) during your internship. Additionally, having clear understandings and expectations for one another is crucial. Therefore, I have developed the following responsibilities for us to honor:

As a student, you have the right to expect that:

- All members of our class community will be treated cordially and respectfully.
- Your submitted work will be evaluated and returned promptly.
- Your work will be graded on criteria that you receive in advance. These criteria will be as objective and transparent as possible, but please acknowledge that grading is ultimately a subjective exercise.
- I will be responsive to questions and concerns, and will respond to emails promptly.

As the course instructor, I have the right to expect that you will:

- Devote the necessary time to fulfill the course requirements. In general, undergraduate courses require 1 hour of outside coursework per every hour spent in the classroom.
- Take responsibility for your learning by staying actively engaged, attending all classes, and consistently checking ELMS.
- If you have an emergency or professional obligation that will impact your attendance, please contact me immediately. We will agree on a make-up assignment that will be due within 2 weeks.
- Maintain professional decorum in all interactions including emails. The use of cell phones (except in case of emergency or when directed) during class time is not permitted, and will negatively affect your participation grade.
- Accept that learning is a process that sometimes feels uncomfortable. The feedback you receive is part of the instructor's professional obligation and is designed to promote growth. I will be glad to discuss pressing concerns about grades (at an agree upon time, NOT via email), but be advised that grades are not to be negotiated.

Acknowledgements: Many thanks to Dr. Dana Grosser-Clarkson, Alice Cook, and Dr. Carolina Napp-Avelli for creating wonderful syllabi to work from and augment.

VIII. Schedule of Activities

The course will focus on pedagogical strategies and key mathematical concepts of middle school mathematics: ratios and proportional relationships, the number system, expressions and equations, geometry, and statistics and probability. Below is a tentative course calendar listed by the daily topic.

Tentative Course Calendar *Note: readings and topics subject to change

Date	Topic	What's Up
Aug 28	*Introductions and Reflections on Our Mathematical Experiences to Inform Mathematics Teaching*	*Read:* NCTM (2014). Progress and Challenge. In Principles to Action: Ensuring mathematical success for all (pp. 1–6). NCTM. Van de Walle, J. A., Bay-Williams, J. M, Lovin, L. H., & Karp, K. S, & (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8.* (2nd ed.). Boston, MA: Pearson. (p. 1-6)
Aug 30	*Current Issues in Math education: CCSS-M*	*Read*: Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: The National Academies Press. (Executive Summary p .1-14) http://www.nap.edu/catalog.php?record_id=9822 Common Core State Standards for Mathematics – Introduction and Standards for Mathematical Practices (pp. 1-8) *Browse* the CCSS-M Standards Document Schmidt, W. H., & Burroughs, N. A. (2013). How the Common Core Boosts Quality and Equality. *Educational Leadership*, *70*(4), 54–58.
Sept 4	*Standards for Mathematical Practice*	*Read:* Mateas, V. (2016). Debunking myths about the standards for mathematical practice. *Mathematics Teaching in the Middle School*, *22*(2), 92 – 99. Assigned chapter(s) on mathematical practice from Koestler, C., Felton, M., Bieda, K., & Otten, S. (2013). Connecting the NCTM Process Standards and the CCSSM Practices. NCTM.

Sept 6	*Establishing classroom norms*	*Read:* Stephan, M. L. (2014). Establishing Standards for Mathematical Practice. *Mathematics Teaching in the Middle School, 19*(9), 532–538. Van de Walle Chapter 1 (p. 6-12) *Skim* Van de Walle p. 123-128 (Addition and Subtraction) *Assignment due*: **Monday September 10th** – Mock Interview
Sept 11	*Mathematical Dispositions and Equity*	*Read:* Berry, R. Q. III. (2004). The Equity Principle through the voices of African American males. *Mathematics Teaching in the Middle School, 10(2),* 100-103. Gresalfi, M. S. & Cobb, P. (2006) Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An international journal, (1)1,* 49 – 57.
Sept 13	*Lesson Planning*	Wilburne, J.M, & Peterson, W. (2007). Using a before-during-after model to plan effective secondary mathematics lessons. *Mathematics Teacher, 101(3),* 209-213. Smith, M.S., Bill, V., & Hughes, E.K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School, 14(3),* 132-138.
Sept 18	*Launching a High-Level task*	Cognitive Demand of Tasks (one-page) Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School, 18*(1), 24–29.
Sept 20	**Back to School Night**	In order to get to know the school community, students in this class are expected to attend the Back to School Night at Bumblebee Middle School on **Thursday September 20, 6:15 pm – 8:00 pm**. This is a very important night to participate in, because it provides you important insight into the school, families, and the unique school community at Bumblebee Middle, particularly with the large number of students who are English Language Learners.
Sept 25	*Effective Questioning Practices*	Herbel-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School, 10(9),* 484-489. Manouchehri, A., & Lapp, D. A. (2003). Unveiling student understanding: The role of questioning in instruction. *Mathematics Teacher, 96(8),* 562-566.

Sept 27	*Orientation at Bumblebee MS*	We will meet at Bumblebee Middle School. Ms. M will provide an orientation. Afterwards we will continue with our regularly scheduled class. **Be sure to bring a government issued photo ID and your fingerprint receipt (cannot be a copy or a photo).
Oct 2	*Eliciting and Responding to Students' Thinking*	Eggleton, P.J., & Moldavan, C.C. (2001). The value of mistakes. *Mathematics Teaching in the Middle School, 7*(1), 42-47. Willingham, J. C., Strayer, J. F., Barlow, A. T. & Lischka, A. E. (2018) Examining mistakes to shift student thinking. *Mathematics teaching in the middle school, 23*(6), 324 – 332. Newton, K.J. & Sands, J. (2012). Why don't we just divide across? *Mathematics Teaching in the Middle School, 17*(6), 340-345. *Skim* Van de Walle p. 135-139 (Division)
Oct 4		Tutoring at Bumblebee MS
Oct 9	*Tailoring Tasks and Multiplying Fractions*	*Read:* McDuffie, A.R., Wohlhuter, K.A., & Breyfogle, M.L. (2011). Tailoring tasks to meet students' needs. *Mathematics Teaching in the Middle School, 16*(9), 550-555. Sanchez, W. B. (2013). Open-ended questions and the process standards. *Mathematics Teacher*, 107(3), 206 – 211. *Skim* Van de Walle p. 128-134 (Multiplication)
Oct 11		Tutoring at Bumblebee MS
Oct 16	*Orchestrating Productive Mathematics Discussions*	Smith, M. S., Hughes, E. K., Engle, R. A., & Stein, M. K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14*(9), 548–556 Rawding, M. R., & Wills, T. (2012). Discourse: Simple Moves That Work. *Mathematics Teaching in the Middle School, 18*(1), 46–51. Reinhart, S.C. (2000). Never say anything a kid can say. *Mathematics Teaching in the Middle School, (5)*8, 478-483.
Oct 18		Tutoring at Bumblebee MS

Oct 23	*Exploring Proportional Reasoning*	*Read:* Lanius, C.S., & Williams, S.E. (2003). Proportionality: A unifying theme for the middle grades. *Mathematics Teaching in the Middle School*, *8*(8), 392-396. Van de Walle – Chapter Eleven: Proportional Reasoning
Oct 25		Tutoring at Bumblebee MS
Oct 30	*Cultural and Linguistic Diversity*	Van de Walle, J. A., Bay-Williams, J. M, Lovin, L. H., & Karp, K. S, & (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8*. (2nd ed.). Boston, MA: Pearson. (p. 58-71) Nikula, J. & Nelson, C. L. (2014). Supporting English Learners: lessons from research. *Mathematics Teaching in the Middle School, 20(1),* 30 – 37. **Lesson Plan for Tutoring Session Due**
Nov 1		Tutoring at Bumblebee MS
Nov 6	*Supporting Students with Learning Disabilities*	Lynch, S. D., Hunt, J. H. & Lewis, K. E. (2018) Productive struggle for all: differentiated instruction. *Mathematics teaching in the middle school*, *23*(4), 194 – 201. Witzel, B. S., & Allsopp, D. (2007). Dynamic concrete instruction in an inclusive classroom. *Mathematics Teaching in the Middle School*, *13*(4), 244-248. Skim Van de Walle p. 77-80 **Lesson Plan for Tutoring Session Due**
Nov 8		Originally scheduled as Tutoring at Bumblebee MS but cancelled due to Parent-teacher conferences
Nov 13	*Algebraic Thinking*	Steele, M. M. & Steele, J. W. (2003). Teaching algebra to students with learning disabilities. *Mathematics Teacher, 96(9)*, 622 – 624. Chan, H. H. (2015). How do they grow? *Mathematics Teaching in the Middle School*, 20(9), 548 – 555. **Audio Analysis Due**
Nov 15		Tutoring at Bumblebee MS – Cancelled due to inclement weather

Nov 20	*Assessment*	Kuper, E. G., & Kimani, P. M. (2013). Responding to Students' Work on a Rich Task. *Mathematics Teaching in the Middle School, 19*(3), 164–171 Wiliam, D. (2016) The Secret of Effective Feedback. *Educational Leadership, 73 (7), 10 – 15.* Van de Walle, Ch 3 – Assessment for Learning (29 – 42)
Nov 27	*Nix the Tricks*	Karp, K. S., Bush, S. B., & Dougherty, B. J. (2015) 12 Math rules that expire in the middle grades. *Mathematics Teaching in the Middle School, 21(4)*, 208 – 215. *Skim:* Cardone, T. *Nix the Tricks.* (2nd Ed). **Lesson Plan for Tutoring Session Due**
Nov 29		Tutoring at Bumblebee MS
Dec 4	*Reflection and Closure*	**Lesson Plan for Tutoring Session Due**
Last Class Dec 6		Tutoring at Bumblebee MS
TBD	Final Exam	The final assessment with be a paper due the date of the scheduled final exam.

Appendix C: Lesson Plan Template

This lesson plan template is designed around the Thinking Through a Lesson Protocol (TTLP).

With your lesson plan, please also submit the mathematical task and any other additional materials (“worksheets”, video, etc.) that you will be using for your lesson. In the materials section please be explicit about what materials (photocopies, manipulatives, etc.) you would like for me to provide.

Before the activity

Time	*Ice breaker Activity*
5 - 10 min	< Description of the ice-breaker activity. This is a way for you to connect with students and build rapport. Please be descriptive. For example, if you are playing “Two Truths and a Lie,” indicate what statements you are going to share with students. >

The Mathematical Task

Description of the mathematical task: <Describe the task and how you will facilitate it. Will students work independently or together? How will students record and report their work? What resources will students have access to?>
Materials needed: <Please distinguish between what you will provide and what you would like for me to provide>

<table>
<tr><td>CCSS:

<Identify the CCSS-M addressed by the task></td><td>Standards for Mathematical Practice:

< Identify the SfMP addressed by the task></td></tr>
<tr><td colspan="2">Prior Knowledge:

< In what ways does the task build on students’ previous knowledge, life experiences, and culture? What definitions, concepts, or ideas do students need to know to begin to work on the task? What questions will you ask to help students access their prior knowledge and relevant life and cultural experiences? ></td></tr>
<tr><td colspan="2">Launch:

< How will you “launch” the task? How will you introduce students to the activity so as not to reduce the demands of the task? What will you hear that lets you know students understand the task? ></td></tr>
<tr><td colspan="2">Solution Methods:

< What are the different ways this task can be solved? Be sure to detail/identify at least two different solution paths. You can insert pictures. ></td></tr>
</table>

Methods: <What methods/solution paths do you think your students will use? Why?>	*Misconceptions and Errors:* < What misconceptions might students have? What errors might they make? Try to identify errors beyond computational errors>
Questions to Focus and Advance: < What questions will you ask to focus student thinking? What questions will you ask to advance students' understanding of the mathematical ideas?>	*Questions to Assess:* < What will you see or hear that lets you know how students are thinking about the mathematical ideas? What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations? >
Engagement: < What assistance could you give or what questions could you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance in solving the task? >	*Orchestrate Discussion:* < How will you orchestrate the group discussion so that you accomplish your mathematical goals? In what order will the (expected) solution paths be presented, and why? How will you handle unexpected student solution paths that work or don't work?>

Closing the lesson: < How will you bring closure to the activity? Revisit your learning goals. Formalize the main ideas of the lesson, helping to highlight connections among strategies or different mathematical ideas. In addition, this is the time to reinforce appropriate terminology, definition, and symbols.>

Closing the session

Time	*Math Game or Activity*
15 - 20 min	< Description of the game. How will you launch the game to students? How will you explain how to play the game? >
Purpose or learning goals of the game: <Why did you choose this game/activity? What mathematical concepts are addressed by the game? >	
Materials needed: < What materials do you need for the game? What will you provide and what do you need me to provide?>	

Appendix D: Lesson Plan Rubric

Item	Unsatisfactory (1 point)	Basic (1.5 points)	Proficient (2 points)
Completeness	The lesson plan is missing a significant amount of requested information.	Some minor details (such as from the ice breaker or the math game) are missing from the lesson plan.	All requested elements of the lesson plan are present. Needed materials are clearly identified.
Cognitive Demand	The selected mathematical task is of low cognitive demand (Procedures without Connections; Memorization).	The selected mathematical task is of high cognitive demand. However, the proposed implementation of the task potentially lowers the demand.	The selected mathematical task is of high cognitive demand (Procedures with Connections; Doing Mathematics). The lesson plan is designed to maintain the high demand of the task.
Learning Goals	The CCSS and Standards for Mathematical Practice identified are inappropriate. OR Learning goals for the math game are not appropriate. OR Learning goals for both the task and the game demonstrate low expectations for students.	CCSS and Standards for Mathematical Practice have been identified, but may not be the “best fit” standards. OR Learning goals for the math game are not clearly articulated. Learning goals for both the task and the game demonstrate high expectations for students.	Appropriate CCSS and Standards for Mathematical Practice are identified for the selected mathematical task. Learning goals for the math game are appropriate and clearly articulated. Learning goals for both the task and the game demonstrate high expectations for students.
Attending to Prior Knowledge	What students know and should be able to do is unclear. OR Overall, the task does not link students’ prior academic learning or personal, cultural, and community assets to new learning.	What students know and should be able to do is described generally OR It is unclear how the planned launch will activate students prior knowledge. For the most part the task links students’ prior academic learning or personal, cultural, and community assets to new learning.	What students know and should be able to do is clearly identified. The planned launch aims to activate students prior knowledge. Overall the task links students’ prior academic learning or personal, cultural, and community assets to new learning.

Solution Methods	Less than two solution methods present. OR A deficit perspective is evident when describing anticipated solution paths and misconceptions/errors.	At least two solution methods are generally described/modeled. Lesson plan articulates which solution path students are most likely to pursue. Identified misconceptions and errors are limited to procedural and computational errors.	At least two solution methods are robustly described/modeled. Lesson plan articulates in depth which solution path students are most likely to pursue and identifies conceptual misconceptions and errors.
Questioning	The planned question sequences are of low cognitive challenge and primarily lead to yes or no responses. Questions may be focused only on procedures and surface level questions, unrelated to the goals of the lesson, and/or may not be suitable for many students.	The planned question are related to the goals of the lesson. The questions are of moderate cognitive challenge OR there is a mix of high and low cognitive challenge.	The planned questions are related to the goals of the lesson. The planned questions provide opportunities for advanced students to extend their understanding and additional supports for students who need it without reducing the cognitive demand. Question sequences elicit student thinking and provide opportunities for mathematical sense making – for problem solving, producing and critiquing extended chains of reasoning, and for building connections between facts and procedures.
Engaging Students in Discussion	Lesson plan attends to elements of student discussion at a surface-level. No attention to how to address unanticipated student work. Lesson closure is absent or does not address the goals of the lesson. OR Discussion and closure consists of only going over the solutions.	Lesson plan generally describes how students will be engaged in discussion around the task, but lacks specificity. Lesson plan has some attention to unexpected student work. The lesson closure connects to the lesson goals.	Lesson plan includes how anticipated solution paths will be sequenced. Plans indicate how all students will be engaged in the discussion of mathematical ideas. Lesson plan attends to the possibility of unexpected student work. There is a lesson closure that consolidates the students' understanding.

Appendix E: Discussion Board Prompts

Enactment Date	Prompt
October 4, 2018	This week was the first tutorial/enrichment at Bumblebee Middle. What grade level did you plan for? What grade level were your students? Share what you learned about their students, both in terms of who they are and what their mathematical strengths are. Be sure to focus on student knowledge rather than deficits.
October 11, 2018	Reflect on Thursday's tutoring session. Write down 3 successes, 2 challenges you encountered, and 1 thing you want to work on (personal/teaching practice) for this upcoming Thursday's session.
October 18, 2018	This week's readings were about student discourse. Which of the strategies from the readings are you, can you, or will you employ during your lessons at Bumblebee Middle?
October 25, 2018	Reflect on Thursday's tutoring session. Write down 3 successes, 2 challenges you encountered, and 1 thing you want to work on (personal/teaching practice) for this upcoming Thursday's session.
November 29, 2018	Reflect back on the semester: What have your students learned, and how do you know? (What is your evidence?) Looking ahead: What are your goals for the final session? Identify goals for students (perhaps learning goals or what you want them to learn) as well as goals for yourself.

Appendix F: Final Paper Prompt

The final assignment for [the methods course] will assess your understanding of teaching and learning middle school mathematics. In this reflection, you will support your understandings with the literature and assignments from this course. It is recommended that you review your blogs, discussion posts, notes you may have taken while reading or during class, as well as your lesson plans and feedback on all assignments.

Format: The paper should be *no more than* 10 pages in length, double-spaced, 12-point Times New Roman font, 1-inch margins. Refer to the APA style guide for in-text citations and reference pages. Your response should attempt to weave together your responses across the various components of the prompt.

Prompt: Reflect on what you have learned in this course and how it has influenced what you now think good mathematics teaching encompasses. Compare and/or and contrast your current understanding of teaching mathematics with your ideas from when you began this class in August. This part of the final exam should demonstrate how your thinking has expanded and changed from the beginning of the course until now. Include content learned in the course, your personal experiences, and your beliefs about teaching. You should use the readings, discussions, assignments, feedback you got on assignments, etc. from the course to support your response. Include quotations/citations from at least 4 different readings we read this semester. Connect the course topics and concepts to the readings, materials, and discussions in the course.

In addition to generally reflecting on the course content, please explicitly respond to the following two prompts:

1. Now that you have completed this course, what type of teacher of mathematics do you want to be? Describe what it entails to teach mathematics for understanding. What will it look like? Sound like? What is the teacher doing? What are the students doing?

→ Compare and contrast your current perspective with your original reflection from the first day of class. How has this course challenged or affirmed your understandings of "good" mathematics teaching?

2. Identify what has been the most impactful aspects of your learning this semester. Stated differently, what have been the most impactful ways you have grown, as both a teacher and learner of mathematics, by taking this class and participating in the enrichment program at Bumblebee Middle School?

→ Clearly describe your areas of growth and provide tangible examples or evidence of your growth

Appendix G: Semi-Structured Interview Protocol for the Initial Interview (Spring 2019)

Introduction: *Thanks for taking the time to chat with me today. My goal in today's interview is to hear your thoughts about TLPL413 in general and the lessons you prepared for your students at Bumblebee Middle specifically. Throughout this interview I will keep asking questions until I think we have a shared understand about what you are saying. This means I might ask questions that seem common-sense! This is not a reflection of you or your responses, rather me doing my due diligence to make sure we have a shared understanding. I will be writing as we are chatting. Don't get nervous! It often means you said something really interesting and I want to hear more about it, but don't want to interrupt you in the middle of a thought. So before you begin, do you have any questions about this interview, or any question in general?*

(Sub-questions are potential follow-up questions to probe participations for elaboration)

Ice-breaker question(s): How are you feeling about the end of the semester? What do you have planned for this summer?

1. Think back to the Fall semester. When preparing your lessons for students at Bumblebee Middle School, what was your process for looking for and selecting mathematical tasks? Walk me through your thought process for planning each week.
    a. What websites or print materials did you utilize?
    b. What are the first things you look for when reviewing tasks?
    c. Did your process change over the course of the semester? Why or why not?

2. How did you decide which tasks to use?
    a. How did the lesson plan rubric influence your task selection and lesson planning?
    b. In class we talked about the cognitive demand of tasks, how did cognitive demand play a role in your decision making?

3. Here are some of your lesson plans from the fall. Why did you select (or create) each of these tasks? Talk me through your decision-making process. (*Note: Prior to the interview I will bring select coursework for the participant to reflect on*).

4. Which of your tasks or lessons from the Fall was your favorite, and why?

5. Did you participate in Terrapin Teachers?
    a. If yes, which Steps did you complete and when?
    b. If yes, how did your experiences in Terrapin Teachers influence your task selection and lesson planning in TLPL413?
    c. If no, prior to TLPL413, what experiences did you have with mathematical tasks and lesson planning?

6. In TLPL413 you have a lot of freedom to select tasks and prepare your lessons. Did you find this freedom liberating or frustrating?
    a. Would you have preferred more structure or pre-selected tasks?

b. What changes would you make to the task selection and lesson planning process in TLPL413?

7. Now I would like for you to examine a task. Pretend that you were searching online and you encountered this task ("The Wheel Shop" retrieved from: http://www.insidemathematics.org). Walk me through your thought process of examining a task – would you choose this task for your students? What might you modify? You can think about your students from tutoring, or some other hypothetical students.

    a. If the interviewee has not yet mentioned cognitive demand, then I will bring up cognitive demand as a criterion to examine.

    b. Prompt interviewee to elaborate on their instructional decisions regarding the selected task.

Appendix H: Semi-Structured Interview Protocol for Second Interview (Fall 2019)

Introduction: *Thanks for taking the time to chat with me today. My goal in today's interview is to hear your thoughts about lesson planning and mathematical tasks now that you are in your internship.. Throughout this interview I will keep asking questions until I think we have a shared understand about what you are saying. This means I might ask questions that seem common-sense! This is not a reflection of you or your responses, rather me doing my due diligence to make sure we have a shared understanding. I will be writing as we are chatting. Don't get nervous! It often means you said something really interesting and I want to hear more about it, but don't want to interrupt you in the middle of a thought. So before you begin, do you have any questions about this interview, or any question in general?*

Ice-breaker question(s): How was your summer break? What are the highlights of your internship thus far?

1. What aspects from your coursework have best prepared you for your work in classrooms?

    a. How has TLPL413 prepared you to work in science classrooms?

2. How has your understanding of selecting and preparing mathematical tasks been impacted since TLPL413?

    a. What has reinforced your understanding? Challenged your understanding?

    b. How does cognitive demand connect to what you are currently learning in your coursework?

3. What opportunities have you had to engage in selecting mathematical tasks and lesson planning in your initial internship?

    a. How do these opportunities compare to the lesson planning you completed for TLPL413?

4. I have five tasks I want you to examine. Take your time to look over each task, and you can do the task if you wish. I would like you to determine the cognitive demand of each task. Walk me through your thinking and how you decide the level of demand for each task.
(See Appendix I)

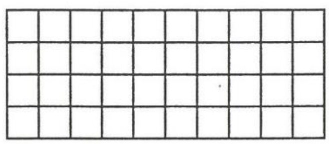
    a. Probe students to explain how they conceptualize cognitive demand.

5. What is the work of being a teacher?
If they ask for clarification: What are the responsibilities of teaching?

    a. What aspects of lesson planning are essential to the work of teaching?

    b. Is task creation or selection/modification essential to the work of a teach?

Appendix I: Mathematical Task Sort from Second Interview

Task	Level of Cognitive Demand	Justification
A. Convert the fraction 3/8 to a decimal and a percent. (p. 13)	Procedures without connections	Performing standard algorithms without additional context or meaning.
B. Shade 6 of the small squares in the rectangle shown below.  Using the diagram, explain how to determine each of the following: (a) the percent of area that is shaded (b) the decimal part of area that is shaded (c) the fractional part of area that is shaded (p. 13, 47)	Doing Math	Asks students to explore the relationships between the various ways of representing fractions. Because the grid is not 10x10, students are required to demonstrate their understanding of fractions, decimals, and percents in novel ways. When the diagram is used to explain the percentage of area, the demand of the task is maintained.
Use algebra tiles to show these multiplications and make a sketch of your model. Write the product. 1. $2x(x - 1)$ 2. $(x + 1)(x + 2)$ 3. $(x - 2)(3x + 3)$ 4. $(x - 3)(x + 3)$ 5. $(2x + 2)(2x - 2)$ 6. $(x + 3)(x + 3)$ C. (p. 96)	Procedures with connections	Students are familiar with using algebra tiles to model operations with polynomials. Students are expected to connect multiple representations of monomial and binomial products.
D. *Part 1* You and the members of your class have just completed a unit on data analysis. As a culminating project for the unit, you and the members of your small group are to design a data collection activity to answer the question "What is Your Favorite — ?" Once you have collected the data, you need to analyze the data and create a graph. *Part 2* The graph assigned in Part 1 has been completed. You and the members of your group have now been asked to make a presentation to the class about your work. The goal of the presentation is to communicate to the class the most important mathematical ideas in your graph. What points should be addressed in your presentation? (p. 110)	Doing Math	The complexity of the task is primarily due to being open-ended. Students must decide what type of graph is most appropriate for their data and determine what the important mathematical ideas are.
E. What are the decimal and percent equivalents for the fractions ½ and ¼ ? (p. 13)	Memorization	Knowing the equivalent forms of specific fractional quantities.

Note: Tasks, classification of cognitive demand, and justification come from Stein, Smith, Henningsen & Silver (2000).

Appendix J: Pre-Service Teacher's Selected Tasks Across All Seven Lessons

	Lesson Number						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Briley	Self (PC)	Self (PC)	Self (PWO)	Joepardylabs.com (PWO)	Self (PWO)	Self (PC)	Self (PC)
Carson	Algebra course (PC)	Self & Kuta Worksheets (PWO)	TPT (PWO)	Cpalms.org & PSAT Prep guide (PWO)	Methods Instructor (PC)	Algebra Course & homeschoolmath.net (PC)	TPT (PC)
Claire	MARS (PC)	Balanced Assessment (DM)	Peer (PC)	Yummymath (PWO)	Self (M)	Inside Mathematics (PC)	Numbers Course (DM)
Elizabeth	NCTM (PC)	Methods Course; College board.org (PC)	Same as L2	Self (PWO)	Same as L4	Methods Course (PC)	Self (PWO)
Grace	Self (PWO)	Self (M)	Self; CommonCore Sheets.com (M)	Self (PWO)	Teacher.org (M)	Methods Course (PWO)	Self (M)
Jake	Self (PWO)	Self (PWO)	NCTM (M)	Self (M)	Same as L4	Kgmathminds.com (M)	Same as L3
Jessica	Pinterest (M)	Methods Course (PC)	Stat Course (PC)	Methods Course (PC)	Peer (PC)	MARS (PC)	Peer (PC)
Mary Jane	Stat Course (PC)	Self; Youtube (PC)	TPT (PC)	Methods Course (PC)	Methods Course (PC)	Peer (PC)	Mathaids.com (PWO)
Sara	NCTM (DM)	NCTM (PC)	Inside Mathematics (PC)	NCTM (PC)	Inside Mathematics (PC)	Balanced Assessment (PC)	NCTM (PC)
Vincent	Self (PWO)	Self (PWO)	Self (PC)	Self (PC)	Warrenkyschools.org (PC)	Self; Google Images (M)	Self (PWO)

For each lesson, the task author is given with the cognitive demand of the task in parentheses. The abbreviations for cognitive demand are as follows: Memorization (M), Procedures Without Connections (PWO), Procedures With Connections (PC) and Doing Math (DM).

Appendix K: Middle Grades Mathematical Tasks by Day

Day	Task	Source
1	Intersecting Lines/ Handshake Problem	http://www.nctm.org/Publications/Teaching- Children-Mathematics/Blog/The-Handshake- Problem/
4	Mango Task	http://illuminations.nctm.org/Lesson.aspx?id=1037
5	Fraction Subtraction Task	Rathouz, M., Rubenstein, R. (2009). Supporting PSTs' learning: A fraction operations task and its orchestration. AMTE Monograph 6- pp. 85-103.
6	Baseball Shop Problem	Howard County Public Schools Math 8 Curriculum
7	In & Out Burger Task Surprising Squares Task	http://robertkaplinsky.com/work/in-n-out-100-x-100/ http://www.lipscomb.edu/uploads/video/wood ard-task.pdf
9	Martha's Carpeting Task/Fencing Task	Stein, Smith, Henningsen, & Silver (2000). (p. 1 - 2)
11	Orangey Task Fraction Division Task	Lappan, G., Phillips, E. D., Fey, J. T. & Friel, S. N. (2014). *Connected Mathematics 3: Comparing and Scaling.*.Boston, MA: Pearson. Boaler & Humphries video; Van de Walle
13	The Marriage Problem	Kuper, E. G., & Kimani, P. M. (2013). Responding to Students' Work on a Rich Task. Mathematics Teaching in the Middle School, 19(3), 164–171.
15	Sharing Pizza Task	Lappan, G., Phillips, E. D., Fey, J. T. & Friel, S. N. (2014). *Connected Mathematics 3: Comparing and Scaling.*.Boston, MA: Pearson.
17	Burning Candles Look-alike Rectangles	Lim, K. H. (2009). Burning the candle at just one end: Using nonproportional examples helps students determine when proportional strategies apply. Mathematics Teaching in the Middle School, 14(8), 492–500. Van de Walle, J. A., Bay-Williams, J. M, Lovin, L. H., & Karp, K. S, & (2014). Teaching student- centered mathematics: Developmentally appropriate instruction for grades 6-8. (2nd ed.). Boston, MA: Pearson.
19	Measure Twice Activity	Zahner, W. C. (2012). ELLs and group work: It can be done well. *Mathematics Teaching in the middle school, 18*(3), 156 – 164.
21	Quadrilateral Task	https://nrich.maths.org/962/note
23	How do they grow?	Chen, H.H. (2015). How do they grow? *Mathematics Teaching in the Middle School*, *20*(9), 548 – 555.
29	The Potato Problem	Created by the course instructor based on the book and movie *The Martian*

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